ABSTRACT: A two-dimensional steady-state solution of coupled turbulent flow and heat transfer in continuous casting of steel billets is solved by a new meshless method. The turbulence is considered by the low-Reynolds number $k-\varepsilon$ model and the solidification effects on the fluid flow in the mushy region are modeled through the Darcy assumption. The meshless method is based on the local collocation with the multiquadrics radial basis functions and explicit time discretisation of the involved mass, energy, momentum, kinetic energy and kinetic dissipation equations. Neither polygonisation nor numerical integration are present in this new method. The velocity-pressure coupling of the incompressible fluid is performed by the fractional step method. The proposed novel numerical method is tested by comparing the results with the CFD commercial code FLUENT. A simulation of the solidification of a spring steel grade in curved mould geometry of the Štore Steel billet caster is performed on a non-uniform node arrangement. The effects of the casting speed on the temperature field are shown. The suitability of this novel approach in simulation of transport phenomena in continuous casting of steel is confirmed.

1. INTRODUCTION

The most important component of the casting machine is the mould, where the liquid steel is poured with high velocity ($\approx 1.0$ m/s) through the submerged entry nozzle (SEN) into the mould [4]. The turbulent flow in the mould region influences the entrapment of the top surface flux layer of the powder and the motion of the inclusion particles. Inclusion particles, exiting the SEN, can be described as impurities of the molten steel or argon bubbles. If the molten powder or inclusions are trapped into the mushy zone (i.e. the solidifying region), the internal cracks and other defects can be generated. In order to prevent these defects and to generally improve the quality of the metal produced, the casting process has to be optimally controlled through the process parameters. Optimal control can be achieved by fully understanding the turbulent fluid flow and other processes inside the mould. The turbulent flow processes are greatly influenced by the geometry (nozzle shape, number of nozzles, etc.) of the SEN, and also its position inside the mould. The analysis of the influence of the process parameters and the geometry parameters of the SEN on the casting process can be very hardly performed by the measurements inside the mould during the casting process. This fact is due to the very high temperature of the liquid steel and inaccessibility of the interior of the process. The numerical models of the continuous casting process are thus increasingly popular, also due to the progress in computer power and model sophistication. There are several advantages of using the numerical methods over the experiments: various physical phenomena can be considered at the same time (coupled mass, momentum, energy and turbulent transport processes together with solidification), the changes in the geometry of the casting machine, the process parameters governing the numerical model can easily be achieved, real thermo-physical material properties can be included, etc.

In this paper, an entirely new meshless Local radial basis function collocation method (LRBFCM) [8] is used to solve the mass, momentum and energy conservation equations which govern the heat and fluid flow model of the continuous casting process. The method was first developed for diffusive problems [8], than for convective-diffusive problems with phase-change [9] and Direct-chill (DC) casting problems for aluminium alloys with material moving boundaries [10]. The method was later applied to the curved geometry of the steel continuous casting machine and to the convection dominated problems, specific for the steel continuous casting processes [11].

The velocity-pressure coupling is solved by the fractional step method [2], where the pressure equation is solved directly through the sparse matrix [6]. The low-Reynolds number two-equation turbulence model [5,13] is used to incorporate the turbulence effects. The numerical procedure was already tested for solving various incompressible turbulent flows,
i.e. 2D channel and backward facing step [12].

2. GOVERNING EQUATIONS

The incompressible turbulent flow of the continuous casting of steel can be described by the following Reynolds time-averaged transport equations for mass, energy and momentum conservation

\[ \nabla \cdot \mathbf{u} = 0, \quad \rho \frac{\partial h}{\partial t} + \rho \nabla \cdot (\mathbf{u} h) = \nabla \cdot \left( \lambda_{\text{eff}} \nabla T \right) \]  

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot (\mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \left[ 2(\mu + \mu_t) \mathbf{S} \right] - C \frac{(1-f_i^2)}{f_L^3} (\mathbf{u} - \mathbf{u}_s) + f \]

with \( \mathbf{u} \), \( p \), \( h \) and \( T \) standing for velocity, pressure, enthalpy and temperature, respectively, and \( \rho \), \( \mu \), \( \mu_t \), \( f_L \) and \( \lambda_{\text{eff}} \) are standing for density, molecular dynamic viscosity, turbulent dynamic viscosity, liquid fraction and effective thermal conductivity, respectively. The third term in Eq. (3) represents the Darcy term, where \( C \) is the morphology constant of the porous medium and \( \mathbf{u}_s \) is velocity of the solid phase. \( \mathbf{S} \) stands for the strain-rate tensor and \( f \) for the buoyancy forces that are neglected in the present model. In Eq. (2), the effective thermal conductivity is defined as \( \lambda_{\text{eff}} = \lambda_i + \lambda_s \), where \( \lambda_i \), \( \lambda_s \) and \( \mu_t \) stand for laminar thermal conductivity, turbulent thermal conductivity and turbulent dynamic viscosity, defined as

\[ \lambda_i = \frac{\mu c_p}{\sigma_i}, \quad \mu_t = \rho c_{\mu} f_{\mu} \frac{k^2}{\varepsilon}. \]

\( c_{\mu} \) and \( f_{\mu} \) in Eq. (5) are closure coefficients of the turbulent model, and \( k \) and \( \varepsilon \) are kinetic energy and dissipation rate, respectively. They are calculated by the following transport equations

\[ \rho \frac{\partial k}{\partial t} + \rho \nabla \cdot (\mathbf{u} k) = \nabla \cdot \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \nabla k \right] + P_k - \rho \varepsilon + \rho D + C \frac{(1-f_i^2)}{f_L^3} k \]

\[ \rho \frac{\partial \varepsilon}{\partial t} + \rho \nabla \cdot (\mathbf{u} \varepsilon) = \nabla \cdot \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + \rho \left( c_{\mu} f_i - c_{2\varepsilon} f_2 \sigma_\varepsilon \right) \frac{\varepsilon}{k} + \rho E + C \frac{(1-f_i^2)}{f_L^3} \varepsilon \]

where \( c_{\mu} \), \( f_i \), \( c_{\varepsilon} \), \( f_1 \), \( c_{2\varepsilon} \), \( f_2 \), \( \sigma_k \), \( \sigma_\varepsilon \) and \( \sigma_T \) are the closure coefficients, and \( D \) and \( E \) are the extra source terms of the low-Reynolds turbulent model [5]. In order to calculate heat and fluid flow on a fixed domain \( \Omega \) with boundary \( \Gamma \), the system of Eqs. (1), (2), (3), (6) and (7) have to be solved.

3. SOLUTION PROCEDURE

We seek the solution of the velocity field, pressure field, temperature field, and \( k \) and \( \varepsilon \) fields at time \( t + t_0 \) by assuming known fields \( \mathbf{u} \), \( p \), \( T \), \( k \) and \( \varepsilon \) at time \( t_0 \) and known boundary conditions at time \( t > t_0 \). The coupled set of mass conservation equation (1) and momentum conservation equations (3) are solved by the fractional step method [2], where the continuity of the mass (1) is considered by constructing the pressure Poisson’s equation. The governing equations are discretized by using the explicit time discretization. This leads to the following algorithm, that is described step-by-step

1) The intermediate velocity components are calculated without the pressure gradient

\[ \mathbf{u}^* = \mathbf{u}^\prime + \frac{\Delta t}{\rho} \left[ -\rho \nabla \cdot (\mathbf{u} \mathbf{u}) + \nabla \cdot \left( 2(\mu + \mu_t) \mathbf{S} \right) \right]^\prime \]
2) The pressure Poisson equation is treated

\[ \nabla^2 p^{n+1} = \frac{\Delta t}{\rho} \nabla \cdot \mathbf{u} \]  

by solving the related sparse matrix [6].

3) The velocity components are corrected by the pressure gradient

\[ \mathbf{u}^{n+1} = \mathbf{u}^* - \frac{\Delta t}{\rho} \nabla p^{n+1} \]  

4) After the solution of the velocity field, the transport equations for energy and turbulence are solved

\[ h^{n+1} = h^n + \frac{\Delta t}{\rho} \left[ -\rho \nabla \cdot (\mathbf{u} h) + \nabla \cdot (\bar{\lambda} \nabla T) \right] \]  

\[ k^{n+1} = k^n + \frac{\Delta t}{\rho} \left[ -\rho \nabla \cdot (\mathbf{u} k) + \nabla \cdot \left( \left( \mu + \frac{\mu_t}{\sigma_k} k \right) \nabla k \right) + P_k - \rho \varepsilon + \rho D + C \left( \frac{1 - f_k^3}{f_k^3} \right) k \right] \]  

\[ \varepsilon^{n+1} = \varepsilon^n + \frac{\Delta t}{\rho} \left[ -\rho \nabla \cdot (\mathbf{u} \varepsilon) + \nabla \left( \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right) + \rho \left( c_{1\varepsilon} f_1 - c_{2\varepsilon} f_2 \varepsilon \right) \varepsilon_k + \rho E + C \left( \frac{1 - f_k^3}{f_k^3} \right) \varepsilon \right] \]  

5) The turbulent viscosity, see Eq. (5), is updated and the solution is set ready for the next time step.

All derivatives of the transport equations are calculated by the LRBFCM, where the collocation is made locally on overlapped sub-domains. On each sub-domain, the scalar function \( \Phi \) (standing for temperature, velocity component, pressure, turbulent kinetic energy and turbulent dissipation rate) is represented over a set of (in general) non-equitable spaced nodes \( p_n; n = 1, 2, ..., N \) in the following way

\[ \Phi(p) \approx \sum_{k=1}^{K} \psi_k(p) \alpha_k, \quad \psi_k(p) = \left[ r_k^2 + c^2 \right]^{1/2}, \]  

where \( \psi_k \) stands for the multi-quadric radial basis shape functions, \( \alpha_k \) for the coefficients of the shape functions, and \( K \) represents the number of the shape functions. In Eq. (15) \( c \) represents the shape parameter and \( r_0 \) the radial distance between two points in the sub-domain. The detailed procedure of calculating the derivatives with the LRBFCM can be found in [8].

4. NUMERICAL EXAMPLE

The solution of the simplified model of the continuous casting process in 2D is presented first. The steady-state solution is shown, approached by a false transient calculation using a fixed time-step of 7.5 \( \times 10^{-4} \) s. The geometry of the simplified casting machine and typical node arrangement is presented in Fig. (1). The length of the mould is 0.8 m, and the length of the computational domain is 1.8 m in order to account for the spray cooling of the billet surface below the mould. The cross-section of the billet is a square. Usually, the material properties are pre-calculated with the JMatPro software, but in this example, the simplified material properties, similar as for the spring steel, are used. These simplifications are used mainly for straightforward validation of the meshless method with the commercial software FLUENT, and in order to reduce the CPU time of the simulations. Although, due to the simple inclusion of physics in the developed meshless numerical method and dimensional flexibility, a very few changes are needed to extend the numerical
algorithm to perform the calculations in three dimensions with more sophisticated physics. However, the preliminary results, where the real curved geometry is considered, are presented in this work as well.

Fig. 1: Casting geometry and typical node arrangement. Left: simplified case for comparison with FLUENT. Right: realistic case. Total number of nodes in both cases is 37867.

4.1 Boundary conditions

The boundary conditions for the velocity and temperature are set as follows:

SEN outlet: The velocity component in the casting direction \( u_y \), \( k \) and \( \varepsilon \) are pre-calculated with the numerical model for the 2D turbulent channel flow. Temperature is constant, equal to the pouring temperature from the tundish.

Billet end: The pressure outlet is prescribed, where the following boundary conditions are used

\[
\begin{align*}
\frac{\partial p}{\partial y} &= 0; \\
\frac{\partial u_y}{\partial y} &= 0; \\
\frac{\partial k}{\partial y} &= 0; \\
\frac{\partial \varepsilon}{\partial y} &= 0; \\
\frac{\partial T}{\partial y} &= 0.
\end{align*}
\] (16)

Top surface (meniscus): At the meniscus, the symmetry boundary conditions are used (free surface flow). Normal derivative of all variables are set to zero, except the vertical velocity \( u_y \) is set to zero.

Stationary wall (SEN): The velocity components, \( k \) and \( \varepsilon \) are set to zero. The Dirichlet boundary condition is used for the temperature. It is set to the casting temperature.

Moving walls: The walls with the solidified steel are moved with the casting velocity along the casting direction. At the walls, where the liquid phase exists, the no-slip boundary conditions for the velocity are set. In the mold, the Robin boundary condition is used, with the surface heat transfer coefficient 2000 W/(m²K). Below the mould, at the secondary cooling system, the heat transfer coefficient is equal to 800 W/(m²K).
4.2 Numerical results

The effects of the casting speed on the temperature field are presented in Fig. (2). The temperature at the surface and at the center of the billet at three different values of the casting velocity: 1.65 m/min, 1.75 m/min and 1.85 m/min. The results obtained by the FLUENT with casting velocity 1.75 m/min are also shown in the same figure. The same low-Re turbulent model, i.e. Launder and Sharma, was set in FLUENT. Despite the physical complexity of the casting process, the results of the LRBFCM (37867 nodes) are in excellent agreement with the FLUENT (33600 cells). Preliminary results of the velocity field, obtained on the curved geometry, are presented in Fig. (3). The same code was used to solve the problem on curved geometry, except the node arrangement was adapted to curved shape of the strand.

Fig. 2: Temperature at the surface and center of the billet along the casting direction at various casting speeds. Solid gray curve – v=1.65 m/min, dashed gray curve – v=1.75 m/min, dash-dot gray curve – v=1.85 m/min. Dashed black curve – FLUENT v=1.75 m/min. Top curves represents the temperature at the billet center.

Fig. 3: Velocity magnitude $u = \sqrt{u_x^2 + u_y^2}$ in (m/s) for curved geometry. Casting speed is $u_s=1.75$ m/min.

$$u = \sqrt{u_x^2 + u_y^2} \text{ in (m/s)}$$
5. CONCLUSIONS

This paper probably for the first time represents the solution of the heat and fluid flow simulation of the continuous casting of steel by a meshless method. Various turbulent models and numerical methods were used in the past to solve such kind of problems. In the present method, we used the low-Re model with the closure coefficients proposed by Launder and Sharma (1974). Other low-Re turbulence models can be implemented in the present novel method [13], however one should care about the compatible boundary conditions for each of them. The numerical solution is based on the local collocation with the radial basis functions for spatial discretization and first order (backward Euler) explicit method for time discretization. Due to its locality and explicit time stepping, the method is very appropriate for parallelization. Non-uniform node arrangement is easily generated, since it does not rely on polygons. The partial differential equations are solved in their strong form, hence no integration is needed. The transition from two-dimensional to three-dimensional cases is quite straightforward. The results were compared with the solution obtained by the commercial software FLUENT on simplified straight geometry, where excellent agreement was achieved. The same meshless code was used to solve a curved geometry case as well. The main advantages of the present numerical approach represent simple coding and no polygonisation.

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6. REFERENCES