SOLUTION OF HEAT TRANSFER AND FLUID FLOW PROBLEMS BY
THE SIMPLIFIED EXPLICIT LOCAL RADIAL BASIS FUNCTION
COLLOCATION METHOD

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Key words: Newtonian fluid flow, primitive variables, natural convection, pressure-velocity coupling, meshless methods, local radial basis function collocation method, multiquadrics.

Abstract. This paper explores the application of the mesh-free local Radial Basis Function Collocation Method (RBFCM) in solution of coupled heat transfer and fluid flow problems. The involved temperature, velocity and pressure fields are represented on overlapping sub-five nodded domains through collocation by using multiquadrics Radial Basis Functions (RBF). The involved first and second derivatives of the fields are calculated from the respective derivatives of the RBF’s. The energy and momentum equations are solved through explicit time stepping. The pressure-velocity coupling is calculated iteratively, with pressure correction, predicted from the local mass continuity equation violation. This formulation does not required solution of pressure Poisson or pressure correction Poisson equations and thus much simplifies the previous attempts in the field. The performance of the method is tested on the classical two dimensional De Vahl Davis steady natural convection benchmark for Rayleigh numbers from $10^3$ to $10^5$ and Prandtl number 0.71.

1 INTRODUCTION

The most commonly used approximate methods for solving the system of partial differential equations (PDEs) in fluid flow problems are the finite difference method (FDM), finite volume method (FVM), the finite element method (FEM), the spectral method (SM) and the boundary element method (BEM). Despite efficiency of enumerated methods in not only fluid flow problems but as well as other physical problems there are some substantial difficulties in applying them to realistic, geometrically complex three dimensional problems. The major problem is in creating a suitable mesh. The meshing is often the most time consuming part of the solution process and is far from being fully automated. However, the rapidly developing
branch of a new kind of meshfree numerical methods, there is no need to create a polygonisation, neither in the domain neither on its boundary, represents a promising technique to avoid the meshing problems (Atluri and Shen, 2002, Chen, 2002, Kansa, 1990).

A number of mesh reduction techniques such as the dual reciprocity boundary element method (Šarler and Kuhn, 1999), meshfree techniques such as the dual reciprocity method of fundamental solutions (Šarler, 2002), meshfree local Petrov Galerkin methods (MLPG) (Atluri and Shen, 2002, Lin and Atluri, 2001) have been developed for transport phenomena and solution of the Navier-Stokes equations. This paper is focused on the simplest class of mesh-free methods in development today, the Radial Basis Function (Buhmann, 2000) Collocation Methods (RBFCM).

The fluid flow problem is generally global problem. In order to solve global problem one needs to solve global matrix (Šarler, 2005, Šarler et al., 2004). Solving matrices for global systems with fine mesh grid or complex geometries can become major numerical problem, therefore completely local scheme for solving fluid flow problems is proposed. Instead of solving the pressure Poisson equation or/and pressure correction Poisson equation (Divo and Kassab, 2007) the much simplified local pressure-velocity coupling (LPVC) algorithm is proposed. The proposed algorithm is tested on classical De Vahl Davis (Davis, 1983) natural convection problem. The results of the method are assessed in terms of streamfunction extreme, cavity Nusselt number, and mid-plane velocity components.

The method represents a local variant of already developed global solution (Šarler, 2005), for coupled heat transfer and fluid flow problems. This local variant was already developed for diffusion problems (Šarler and Vertnik, 2006), convection-diffusion solid-liquid phase change problems (Vertnik and Šarler, 2006) and subsequently successfully applied in industrial process of direct chill casting (Vertnik et al., 2006). In this paper the spectra of physics coped is extended to solution of coupled mass, energy and momentum equations.

2 GOVERNING EQUATIONS

The steady-state natural convection problem is described with three coupled PDEs and Bussinesq approximation. The PDEs are mass, momentum and energy conservation where all material properties are considered to be constant. The equations are given as

\[ \nabla \cdot \mathbf{v} = 0, \]  
\[ \nabla \cdot (\rho \mathbf{v} \cdot \mathbf{v}) = -\nabla P + \nabla \cdot (\mu \nabla \cdot \mathbf{v}) + \mathbf{f}, \]  
\[ \nabla \cdot (\rho c_p T \mathbf{v}) = \nabla \cdot (\lambda \nabla T), \]  
\[ \mathbf{f} = \rho [1 - \beta (T - T_{ref})] \mathbf{g}. \]

with \( \mathbf{v}, P, T, \lambda, c_p, \mathbf{g}, \rho, \beta, T_{ref}, \mu, \) and \( \mathbf{f} \) standing for velocity, pressure, temperature, thermal conductivity, specific heat, gravitational acceleration, density, coefficient of thermal
expansion, reference temperature for Bussinesq approximation, viscosity and body force, respectively. The problem is solved on a fixed domain $\Omega$ with boundary $\Gamma$ where Dirichlet and Neumann boundary conditions for temperature might be used and Dirichlet boundary conditions for velocity are used.

3 SOLUTION PROCEDURE

In order to solve the problem, the time dependent variant of equations (2) and (3) is employed. The explicit time scheme with finite difference approximation is adopted to calculate the time derivative. To solve the Navier-Stokes equations (1) and (2) are solved iteratively. The LPVC algorithm, where pressure correction is estimated from local mass continuity violation, is used to drive intermediate velocity towards divergence-free velocity.

In the first step the velocity is estimated from the discretized form of equation (2)

$$\hat{v} = \mathbf{v}_0 + \frac{\Delta t}{\rho} \left[ -\nabla P_0 + \nabla \cdot \left( \mu \nabla \mathbf{v}_0 \right) + \mathbf{f}_0 - \nabla \cdot (\rho \mathbf{v}_0 \cdot \mathbf{v}_0) \right],$$

where $\hat{v}$ denotes velocity at time $t_0 + \Delta t$, $\mathbf{v}_0$, $P_0$ denotes velocity and pressure at time $t_0$ and $\Delta t$ denotes time-step. The calculated velocity $\hat{v}$ does not satisfy the mass continuity equation (1) in general. In order to couple mass continuity equation and momentum equation, the iteration is used where the first iteration velocity and pressure are set to

$$\mathbf{v}^{m} = \hat{v}, \quad P^{m} = P_0; \quad m = 1,$$

where $m$ stands for iteration index. To project the velocity into the divergence free space the correction term is added

$$\nabla \cdot (\mathbf{v}^m + \tilde{v}) = 0 \quad \rightarrow \quad \nabla \cdot \mathbf{v}^m = -\nabla \cdot \tilde{v},$$

where $\tilde{v}$ stands for velocity correction. Velocity correction is affected only by effect of pressure correction

$$\tilde{v} = -\frac{\Delta t}{\rho} \nabla \tilde{P},$$

where $\tilde{P}$ stands for pressure correction. The pressure correction Poisson equation is constructed by applying the divergence to equation (8)

$$\nabla^2 \tilde{P} = \frac{\rho}{\Delta t} \nabla \cdot \mathbf{v}^m.$$

Instead of solving the equation (9) globally with proper pressure correction boundary conditions (Divo and Kassab, 2007) the pressure correction is assumed to be linearly related to the Laplace of pressure correction, therefore in the second step, the pressure correction is calculated as
\[ \bar{P} \approx L^2 \nabla^2 \bar{P} = L^2 \frac{\rho}{\Delta t} \nabla \cdot \mathbf{v}^m, \]  
\( \text{(10)} \)

where \( L \) stands for characteristic length. The assumption (10) enables solving the problem completely locally. In the third step, the intermediate pressure and velocity are corrected as

\[ P^{m+1} = P^m + \beta \bar{P}, \]
\[ \mathbf{v}^{m+1} = \mathbf{v}^m - \beta \frac{\Delta t}{\rho} \nabla \bar{P}, \]  
\( \text{(11)} \)

where \( \beta \) stands for relaxation parameter. If the criteria

\[ \nabla \cdot \mathbf{v}^{m+1} < \varepsilon_v, \]
\( \text{(12)} \)

is not met than the iteration returns back to the equation (10), else pressure-velocity iteration is completed and the calculation proceeds to the next step.

The fourth step is to solve the time dependent variant of energy equation (3)

\[ T = T_0 + \frac{\Delta t}{\rho c_p} \left[ \nabla \cdot (\lambda \nabla T_0) - \nabla \cdot (\rho c_p T_0 \mathbf{v}_0) \right], \]  
\( \text{(13)} \)

where \( T_0 \) and \( T \) denote temperature at time \( t_0 \) and \( t_0 + \Delta t \). The steady-state is achieved when the criteria

\[ \frac{|T - T_0|}{|T_0|} < \varepsilon_T; \quad T_0 \neq 0 \]
\[ T < \varepsilon_T; \quad T_0 = 0 \]  
\( \text{(14)} \)

is met in all nodes. If the criteria (14) is not met, the body force is updated and calculation returns back to the equation (5). The simulation flowchart is presented in the Figure 1.
4 RADIAL BASIS FUNCTION COLLOCATION METHOD

The pressure, velocity and temperature fields are interpolated on the same grid points by RBFCM where Hardy’s multiquadrics are used as basis functions. The arbitrary function $\theta$ is represented in local sub-domain as

$$\theta(\mathbf{p}) \approx \sum_{n=1}^{N} \Lambda_n(\mathbf{p}) \alpha_n,$$

with $\mathbf{p}$, $\Lambda_n$, $\alpha_n$ and $N$ standing for position vector, basis function, collocation coefficient and number of collocation points, respectively. Hardy’s multiquadrics basis functions are defined as

$$\Lambda_n(\mathbf{p}) = \sqrt{r_n^2(\mathbf{p}) + c^2 r_0^2}, \quad r_n^2 = (\mathbf{p} - \mathbf{p}_n) \cdot (\mathbf{p} - \mathbf{p}_n),$$

where $c$ represents a dimensionless shape parameter. The scaling parameter $r_0^2$ is set to the maximum nodal distance in the sub-domain. The collocation coefficients are obtained from a collocation condition in the nodal points where equation (15) must hold. In case when the number of nodes is the same as number of terms in series (15), the system simplifies to

$$\left[ \begin{array}{c} \Lambda_{11} & \cdots & \Lambda_{1N} \\ \cdots & \cdots & \cdots \\ \Lambda_{N1} & \cdots & \Lambda_{NN} \end{array} \right] \left[ \begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_N \end{array} \right] = \left[ \begin{array}{c} \theta_1 \\ \vdots \\ \theta_N \end{array} \right],$$

where $\Lambda_{ni} = \Lambda_n(\mathbf{p}_i)$. Solving the matrix equation (18) gives the collocation coefficients $\alpha_n$.

Spatial derivatives of the function $\theta$ can be easily obtained by deriving the equation (15)

$$\frac{\partial}{\partial p_\sigma} \theta(\mathbf{p}) \approx \sum_{n=1}^{N} \frac{\partial}{\partial p_\sigma} \Lambda_n(\mathbf{p}) \alpha_n,$$

$$\frac{\partial^2}{\partial^2 p_\sigma} \theta(\mathbf{p}) \approx \sum_{n=1}^{N} \frac{\partial^2}{\partial^2 p_\sigma} \Lambda_n(\mathbf{p}) \alpha_n,$$

where $p_{\sigma=x,y}$ stands for Cartesian coordinates. With equations (19) and (20) all necessary derivatives to construct the involved divergence, gradient and Laplace operators can be calculated. The integral of function $\theta$ over $p_\sigma$ can be evaluated as well

$$\int \theta(\mathbf{p}) dp_\sigma = \sum_{n=1}^{N} \int \Lambda_n(\mathbf{p}) dp_\sigma,$$

In this paper only the simplest sub-domain of five collocation points is used with the overlapping collocation sub-domain strategy. The described collocation method and sub-domain selection is schematically presented in Figure 2, where five points collocation sub-domain is used to approximate first and second spatial derivatives in central grid point.
At boundary collocation points with the Neumann boundary conditions the derivative instead of function value is known. In such points the equation (17) is replaced with

$$\frac{\partial}{\partial p_\sigma} \theta(p_i) = \sum_{n=1}^{N} \frac{\partial}{\partial p_\sigma} \Lambda_n(p_i) \alpha_n,$$

where index $i$ stands for index of point where derivative is known.

5 NUMERICAL EXAMPLES

The classical De Vahl Davis (Davis, 1983) natural convection problem is considered for benchmarking purposes. The domain of the problem (Figure 3) is a closed square cavity filled with air (Prandtl number=0.71) with differentially heated vertical walls ($\Delta T = T_H - T_C$) and isolated horizontal walls.

With constant initial temperature, pressure and velocity set to zero, the steady-state is achieved through time transient. All results are stated in Cartesian coordinates and standard
dimensionless form (Wan et al., 2001)

\[ x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L}, \quad u = \frac{\bar{u} L \rho c_p}{\lambda}, \quad v = \frac{\bar{v} L \rho c_p}{\lambda}, \quad \Psi = \frac{T - T_c}{T_H - T_c}, \quad \tau = t \frac{\lambda}{\rho c_p L^2}, \] (23)

where \( x, y \) stand for dimensionless coordinates, \( u, v \) stand for dimensionless horizontal and vertical velocity components, \( \Psi \) stands for dimensionless temperature and \( \tau \) stands for dimensionless time. Prandtl and Rayleigh numbers are calculated from equations.

\[ \text{Pr} = \frac{\mu c_p}{\lambda}, \] (24)

\[ \text{Ra} = \frac{g \beta \Delta T L^3 \rho c_p}{\lambda \mu}, \] (25)

where \( \Delta T \) stands for maximum temperature difference and \( L \) stands for enclosure length. Non-permeable and due to consideration of viscous fluid no slip boundary conditions are adopted on a whole boundary \( \Gamma \)

\[ \mathbf{v} \Gamma = 0. \] (26)

The results are presented in terms of stream functions and temperature contours in Figure 4 and mid-plane velocities in Figure 5, respectively. The temperature contour plot step is 0.05 for all cases while streamfunction contour plot step is 0.1 for \( \text{Ra} = 10^3 \), 0.5 for \( \text{Ra} = 10^5 \), 1 for \( \text{Ra} = 10^5 \) and \( \text{Ra} = 10^6 \) and 5 for \( \text{Ra} = 10^7 \) and \( \text{Ra} = 10^8 \). For possible straightforward comparison the mid-plane velocity numerical values are stated in Table 3, as well
Comparison of the results (Table 1) is done for maximum mid-plane velocities, mid-point stream function value and average Nusselt number on hot or cold wall where the results of present study are compared with Davis (1983) (a), Sadat and Couturier (2000) (b), Wan, et al. (2001) (c) and Šarler (2005) (d). The Streamfunction is calculated by integrating velocity component

$$\psi(x, y) = \int u(x, y) dy,$$  
(27)
Figure 5: Mid-plane velocities

where $\psi$ stands for streamfunction. The Nusselt number is calculated locally on the support of five collocation points by equation

$$\text{Nu}(x, y) = -\frac{\partial \Psi(x, y)}{\partial x} + u(x, y)\Psi(x, y).$$

All results are compared with simulation performed on 41x41 (with $N_{\text{max}} = 1677$), 81x81 (with $N_{\text{max}} = 6557$) and 101x101 (with $N_{\text{max}} = 10197$) grid sizes, where $N_{\text{max}}$ stands for number of grid points. Additional method check is done by considering the time dependent mass continuity equation (1) globally, to check the numerical mass leakage. The following equation is implemented

$$\rho \rho_{t} + \rho \nabla \cdot \mathbf{v} = 0,$$

where $\rho$ and $\rho_{t}$ stands for time-step index and global density. The global density change is introduced as
\[ \Delta \rho = \left| \rho_t - \langle \rho \rangle_t \right|, \]  

where \( N_t \) stands for number of time-steps. The relative density change \( \Delta \rho / \rho_0 \) due to numerical error during the calculation on grid size 101x101 for different Rayleigh numbers are stated in Table 2.

The time-step criteria \( \varepsilon_t < 10^{-5} \) is used for all cases while pressure-velocity iteration criteria varies for different Rayleigh number (Table 2). The time-step varies for different Rayleigh numbers, as well. For all cases relaxation parameter is set to same value as dimensionless time-step. The number of pressure-velocity iterations, time-steps and actual calculation time for different Rayleigh numbers and grid sizes are also stated in Table 2. All cases are calculated with RBF shape parameter \( c = 30 \).

### Table 1: Results comparison

<table>
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<tr>
<th>Ra</th>
<th>( v_{\text{max}} )</th>
<th>( y )</th>
<th>( u_{\text{max}} )</th>
<th>( x )</th>
<th>( \bar{N}_t )</th>
<th>( \psi_{\text{mid}} )</th>
<th>ref / discretization</th>
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<td>3.679</td>
<td>0.179</td>
<td>3.634</td>
<td>0.813</td>
<td>1.116</td>
<td>1.174</td>
<td>(a)</td>
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<td>3.686</td>
<td>0.188</td>
<td>3.489</td>
<td>0.813</td>
<td>1.117</td>
<td>1.165</td>
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<td>3.566</td>
<td>3.544</td>
<td>3.931</td>
<td>0.825</td>
<td>1.101</td>
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Numerical implementation was done in C++ programming language and compiled with Intel C++ 9.1 compiler. The LAPACK routines are used to solve the LU decomposition. The parallelisation is implemented with OpenMP library with maximum $1.85 \times$ speedup factor achieved for two CPU cores. All calculations were done on a laptop computer Toshiba Satellite 100, with duo core Intel 2.16 GHz processor and 1 Gb of RAM. Due to only one step pressure correction assumption the algorithm needs small number of calculations per iteration cycle and that makes the algorithm fast and robust. Good agreement with other methods at given range (Rayleigh number from $10^3$ to $10^8$ and grid with maximum 101x101 grid points) is achieved with proposed algorithm. High deviation from results (c) at high Rayleigh number is due to grid selection. The present results are calculated on uniform grid when the results (c) are calculated on Gauss-Lobatto grid, which is much more reasonable grid selection due to the highest velocities near the boundary layer (Figure 5). The effect is more intense with higher Rayleigh number and so the deviation is higher with higher Rayleigh numbers.
6 CONCLUSIONS

This paper explores the simplified local RBFCM approach to calculation of the coupled heat transfer and fluid flow problems with entirely local pressure correction. The algorithm is very simple to numerically implement, fast and robust. Due to completely explicit time stepping the algorithm can be efficiently parallelized to exploit the full power of multi core processors.

Further work will be focused on more complex geometric situations and more complex physical models (porous media, solidification, ... ), which seem to be quite simple to numerically implement in the present context. Another future issue is the investigation on adaptive time dependent grid to enhance the accuracy and to avoid the eventual stability problems.

A remark should be made in the sense that there are possible difficulties when working with finer grids than in present paper. The proposed algorithm includes only few surrounding points to calculate pressure correction. The present pressure correction calculation (calculated only from closest neighbouring points) may not be efficient enough when working with finer grids (200x200 or finer) and high Rayleigh numbers (more than $5\cdot10^7$). Possible upgrade is to include wider domain of points in the pressure correction calculation. These topics are subject of further investigations.

7 REFERENCES


