An identification and control strategy for a liquid composite molding process

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Abstract

Liquid Composite Molding (LCM) is a manufacturing process that involves the injection of resin into a mold containing fiber preform and the subsequent curing. Control of the resin injection is an obvious way to increase process performance. Variability in the properties of the fibers makes open-loop approaches problematic. This paper presents a controller for a 1-D analog of the LCM process. The purpose is to demonstrate that a straightforward estimation and control scheme can adapt to unmodelled variations in the properties of the mold, and enforce a prescribed filling behavior. The scheme is applied to a simulation of an LCM process. © 1998 Elsevier Science Inc. All rights reserved.

1. Introduction

Liquid Composite Molding (LCM) is a manufacturing process that involves the injection of resin into a mold containing fiber preform and the subsequent curing. In terms of material properties, such as strength-to-weight ratio, LCM products are very competitive with more traditional fabrication materials such as steel. Their wide scale use, however, is hampered by the fact that the process cycle time is too long for industries that require high levels of productivity, e.g., automobile manufacturing.

An obvious way of cutting down on the cycle time of an LCM process is to use process control. An easy and significant point to control would be the resin injection process, with the objective of obtaining a desired filling pattern and time to promote optimum curing.

The first step in an injection control strategy is determining a suitable open-loop control. The development of this control would involve the use of an off-line filling simulation to identify the optimum placement and operation of injection ports. The effectiveness of this step rests on a sound model of the injection process. On first sight this is not a problem, since many successful and accurate LCM injection process models have been presented in the literature [1–5]. A key parameter in the successful implementation of a model, however, is the permeability of the mold. In

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a real process there are uncertainties associated with this parameter that will limit the applicability of any open-loop control. This list of uncertainties includes:

1. **Preform placement:** In many processes it is not possible to place the preform so that it exactly fits the mold. Usually there is a thin gap between the preform and the mold wall. This gap can lead to the enhancement of resin flow along the edges of the mold; a phenomenon referred to as “race tracking.” In some situations race tracking has a negative effect on the process, since it can lead to the entrapment of air voids (“dry spots”) that are a serious flaw in the finished product. In other cases – if well understood – race tracks can be used to promote the filling of the part. In either case the effect of race tracking can be modelled in a filling simulation, by prescribing an enhanced permeability along the mold edges. However, the degree of enhancement is critically dependent on the gap width \(d\) – a value which is very difficult to control in an industrial setting.

2. **Preform anisotropy:** In many cases, the natural directionality of the preform will lead to an anisotropic permeability. Although it is easy to characterize the anisotropic behavior for a given preform, in an industrial setting it would be expected that there could be significant differences between one preform and the next, due to variations in “lay-up” – the process by which the preform is put into the mold.

3. **Materials:** The LCM injection process is sensitive to the fiber and resin characteristics. In an industrial setting it is expected that due to changes in fiber and resin suppliers, or to low tolerances in the fiber production process, or to resin aging, these characteristics will change over the life of the LCM process.

4. **Geometry:** The permeability of a given fiber is usually estimated from an experiment that involves the center injection of a fiber disk. The permeability is then calculated by comparing the measured filling front movement with a known analytical solution \(7,8\). A typical industrial part, however, would not be flat; in particular, it would include bends and curves. Due to the deformation of the fabric and the possible disalignment of the preform the local permeability in the vicinity of a bend or curve would be different from the bulk value \(9\). Due to variations in process operation it may not be possible to characterize this difference a priori.

The bottom line is that an open-loop control strategy that performs well in simulations, or in laboratory experiments, may well be unacceptable under industrial conditions. Accounting for the process uncertainties listed above requires augmenting the open-loop control with feedback. The resulting closed-loop controller makes appropriate adjustments in real-time to account for unmodelled variations in the mold characteristics.

The objective of this paper is to initiate the development of an adaptive control strategy for LCM molding. The starting point will be to consider parts in which the flow is essentially one-dimensional. Emphasis will be placed on developing methods that can deal with cases where the permeability of the mold is an a priori unknown function of space (e.g., a curved part). Extension to 2-D geometries, and time-dependent flows (such as those due to absorption of resin into the fibers), will be the subject of further research.

2. **The 1-D flow model**

The 1-D LCM problem posed here may be envisioned as follows: A long tube of constant, unit, area is packed with fibers. The permeability of the packed fibers is not necessarily uniform along the length of the tube, for the reasons given in Section 1. A pressurized tank containing a resin is attached to the start of the tube (i.e., at \(x = 0\)). The pressure in the tank is the pressure at the tube inlet, \(P_0(t)\), which can be controlled exactly. Note that this pressure is typically much larger than the hydrostatic pressure, and hence effects of gravity can be neglected \(4\). In addition, the flow
rate out of the tank, \( Q_0(t) \), can be measured exactly (for example, by measuring the level of fluid in the tank). At time \( t = 0 \), resin begins to flow into the tube. The flow of resin is assumed incompressible, and the advancing interface between wet and dry fibers is assumed planar. This interface will also be referred to as the front, and its position denoted by \( X(t) \). The pressure distribution in the resin, between \( x = 0 \), and \( x = X(t) \), is denoted \( P(x, t) \). The pressure in the unfilled regions, and at the front, is assumed to be identically zero. The objective of this study is to vary \( P_0(t) \) so that the motion of the front follows a prescribed behavior, \( X_{\text{des}}(t) \). Fig. 1 shows a schematic of the 1-D problem.

2.1. Equations of motion

The pressure field is governed by the unforced field equation,

\[
\frac{\partial}{\partial x} \left( \lambda(x) \frac{\partial P}{\partial x} \right) = 0
\]

subject to,

\[
P(0, t) = P_0(t), \quad P(X(t), t) = 0.
\]

Here \( \lambda(x) = k(x)/\mu \), where \( k \) is the permeability of the fibers, and \( \mu \) is the viscosity of the resin. For convenience of presentation, the parameter \( \lambda \) will be referred to as the conductivity. As discussed in Section 1, the conductivity is not well known a priori – for a number of reasons – and may vary along the length of the tube.

The motion of the boundary is given by,

\[
\frac{dX}{dt} = - \left( \lambda(x) \frac{\partial P}{\partial x} \right) \bigg|_{x=X(t)}.
\]

Fig. 1. The 1-D LCM problem.
Eqs. (1) and (3) are solved, with the given boundary conditions, Eq. (2), to obtain,

$$P(x, t) = P_0(t) \left( \frac{A(x) - A(X(t))}{A(x)} \right),$$

where $A$ is the total conductivity, defined by,

$$A(x) := \left[ \int_0^x \frac{d\xi}{\lambda(\xi)} \right]^{-1}.\tag{5}$$

The corresponding interface velocity is,

$$\frac{dX}{dt} = A(X(t))P_0(t).\tag{6}$$

By the assumptions of incompressibility and unit area, the flow rate into the tube, $Q_0$, must be exactly equal to the interface velocity, $dX/dt$. That is,

$$Q_0(t) = A(X(t))P_0(t).\tag{7}$$

2.2. Simulation

The objective of this paper is to develop a control strategy that can handle unmodelled variations in the preform properties. This strategy is tested by controlling a simulation of a filling process. In the simulation the variations in the conductivity $A(x)$ are prescribed. This prescription will, however, remain unknown to the control algorithm. Since only the interface motion is of interest, it suffices to integrate Eq. (6), with the value for $P_0(t)$ provided by the control algorithm. Then, if desired, the full pressure field can be recovered by substituting the resulting $X(t)$ into Eq. (4).

Eq. (6) is a state equation for $X(t)$, with $P_0(t)$ playing the role of a forcing term, or a control input. $A(X)$ may be obtained by explicitly integrating Eq. (5) (symbolically or numerically). Alternatively, $A(X(t))$ may be included in the state vector. The second approach has been taken here, with the state equation for $A$ found by setting $x = X(t)$ in Eq. (5), and differentiating the result with respect to time, to yield, $A = -A^2X/\dot{\lambda}(X)$. Then, using Eq. (6) to substitute for $X$ gives the first-order time-varying system,

$$\frac{d}{dt} \left[ \frac{X}{A} \right] = \left[ \frac{AP_0(t)}{-AP_0(t)/\dot{\lambda}(X)} \right],\tag{8}$$

where $\dot{\lambda}(x)$ is prescribed, and $P_0(t)$ is either specified a priori (open-loop control algorithm), or calculated as the simulation progresses (closed-loop control algorithm).

Note that for the natural choice of initial condition, $X(0) = 0$, the total conductivity, $A(0)$, as given in Eq. (5), is undefined. Therefore, the simulation must be started at a small distance from the beginning of the tube.

3. Identification and control

For a specified function $\dot{\lambda}(x)$, given any desired $X(t)$ satisfying $X(0) = 0$, a corresponding $P_0(t)$ can be found from Eq. (6). Then, if the actual $\dot{\lambda}(x)$ matches the assumed specification, applying this open-loop control pressure will yield the desired interface motion. However, if the actual $\dot{\lambda}(x)$ is not as assumed, the resulting interface motion will not follow the expected profile. In such cases,
achieving the designed behavior requires feedback of a measured quantity, and the natural feedback for this problem is the measured flow rate, \( Q_0(t) \). The resulting control design has three parts. First, the total conductivity is obtained from \( P_0(t) \) and \( Q_0(t) \). Next, that information is used to generate an estimate of \( \lambda(x) \). Finally, a control based on this \( \lambda \) is calculated and applied, until the estimate is updated.

Though several approaches are possible to the 1-D problem, the most practical, from the point of view of eventual implementation, is an adaptive, discrete-time controller. Here, the uncertain system parameters are updated at a regular interval, denoted \( \Delta \tau \), and the control for the next interval is based on the update. That is, at time \( t_i := i\Delta \tau \) the parameter estimate is updated, and the control recomputed. The resulting control is applied over the interval \((t_i, t_{i+1}]\).

It is assumed below that the raw measurement, \( Q_0 \), is available continuously, and that the linear pressure profile, \( P_0(t) = A_0 + B_0 t \), can be applied in continuous time (note that the values of \( A_0 \) and \( B_0 \) will typically change from one control interval to the next).

3.1. Estimating the conductivity

At any time, \( t \), the total conductivity is,

\[
A(X(t)) = \frac{P_0(t)}{Q_0(t)}.
\]  

From this, it is desirable to extract as much information about the actual conductivity distribution as possible. Without making any further assumptions about the allowable variations in conductivity, this means determining the effective conductivity over the interval travelled by the front since the last parameter update. The effective conductivity is defined to be the constant value \( \lambda_{e} \) that would yield the observed change in total conductivity, \( \Delta A_i = A_i - A_{i-1} \) over the interval \( \Delta X_i = X_i - X_{i-1} \). The effective conductivity is given by,

\[
\lambda_e = -\frac{A_i A_{i-1}}{\Delta A_i / \Delta X_i}.
\]  

This can be seen to be a discrete approximation to the exact value for \( \lambda(x) \), which is,

\[
\lambda(x) = -\frac{A^2(x)}{dA(x)/dx}.
\]  

For the 1-D problem, \( X_i \) is found from \( \int_{0}^{t_i} Q_0(\tau) \, d\tau \) (where the value is exact, by the assumption that \( Q_0(t) \) is available continuously), and \( A_i \) is, following Eq. (9) given by \( \frac{P_0(t_i)}{Q_0(t_i)} \). Though Eq. (10) is given in terms of conductivity, it may be more easily understood in terms of the resistivity, \( \rho(x) := 1/\lambda(x) \). Then, the mean resistivity over the interval \((X_{i-1}, X_i)\) is \( \bar{\rho}_i = (1/\Delta X_i) \int_{X_{i-1}}^{X_i} \rho(x) \, dx \), and the effective conductivity is \( \lambda_e = 1/\bar{\rho}_i \).

Once a measurement has been made, and the estimate of the conductivity distribution \( \lambda(x) \) updated, a control must be calculated for the next interval. Of course, the front has yet to reach those points, and so no conductivity data is available on which to base the calculation. Therefore it is always necessary to extrapolate the conductivity estimates farther along the tube than the current interface position. The extrapolation may be based on a priori knowledge of the possible system behaviors, but without such knowledge, the most straightforward approach is to apply the effective conductivity from the previous interval to the current interval. The scheme is summarized in Fig. 2, which shows the situation at \( t_i \). The interface is at position \( X_i \), and the associated total conductivity, \( A_i \), is known exactly. The tube can be segmented into intervals, each corresponding to a control interval, \( \Delta \) (note that the segments typically are not of equal length). The effective conductivity, \( \lambda_e \), in \((X_{i-1}, X - i]\), the interval just traversed, is calculated from Eq. (10). Then this value is used as an estimate, \( \lambda_{e+1} \), for the upcoming interval.
3.2. Controlling the interface

The objective of the 1-D LCM control problem is tracking a specified interface motion. That is, driving the actual front position \( X(t) \) so that it follows a given desired profile, \( X_{\text{des}}(t) \). The control variable is the driving pressure \( P_0(t) \). The governing equations and their solution, given in Section 2.1, can be used to derive the open-loop pressure profile corresponding to any desired front position profile. The necessary formula is given by Eq. (6), rewritten as,

\[
P_0(t) = \frac{X_{\text{des}}(t)}{A(X_{\text{des}}(t))}.
\]

Of course, this control requires knowledge of \( A(x) \), and so cannot be implemented without some kind of estimation scheme.

The estimation scheme used is as described in Section 3.1. The control will be generated by assuming a constant \( \lambda \) over each control cycle, denoted for the control cycle \( (t_i, t_{i+1}] \) by \( \lambda_{i+1} \). As explained above, the method used here is to set \( \lambda_{i+1} = \hat{\lambda}_i \). This step could be made more elaborate, with polynomial fits, or parametric models for example. However, effective use of such techniques requires a priori modelling of the conductivity profile, which is beyond the scope of this study.

If this choice for \( \hat{\lambda}_{i+1} \) were exactly correct for every interval, then a continuous, piecewise-linear, approximation to \( X_{\text{des}}(t) \) would be obtained by applying the linear pressure profile, given by appropriate substitution into Eq. (12),

\[
P_0(t) = \frac{\bar{V}_i(\hat{\lambda}_i)(X_i + \bar{V}_i(t - t_i))}{\Delta \lambda_i}, \quad t \in (t_i, t_{i+1}],
\]

where \( \bar{V}_i = (X_{\text{des}}(t_{i+1}) - X_i) / \Delta r \). It may be easily verified that the resulting \( X(t) \), is piecewise-linear, and satisfies \( X(t_{i+1}) = X_{\text{des}}(t_{i+1}) \), that is, it exactly matches the desired profile at the endpoints of each control interval.

Referring to Eq. (6), see that it is the total conductivity that governs the interface velocity, rather than the instantaneous value at the front. This is good news from the standpoint of estimation and control, since the smoothing behavior of the integral in Eq. (5) tends to reduce the effect of uncertainty in \( \lambda \). This effect also reduces the benefits to be gained by using a priori models of the spatial dependence of \( \hat{\lambda} \).

Fig. 2. The estimation and prediction scheme.
4. Numerical results

The simulation technique described in Section 2.2 was applied to the filling of a tube with length \( L = 0.2 \) m. The fluid properties were selected to give a nominal conductivity of unity. The state equations, Eq. (8), were integrated using a fourth-order, adaptive step-size, Runge–Kutta–Fehlberg algorithm, as implemented in the MATLAB routine \texttt{ode45} [10]. A maximum stepsize of 0.01 was enforced to guarantee a large number of simulation steps in every control cycle. The interface position was initialized to 0.001 m, and the total normalized conductivity was set to 1000, corresponding to an effective conductivity of one over the initial small interval. The desired interface motion was chosen to be linear,

\[
X_{\text{des}}(t) = Vt
\]

with a constant velocity \( V \) of 0.01 m/s.

The estimation and control scheme given by Eqs. (10) and (13) was applied to several test cases, each displaying a different conductivity distribution. Physically, the conductivity must be a non-negative quantity. This constraint is reflected in the choice of multiplicative perturbations applied. The cases were as follows:

1. Nominal conductivity
   \( \lambda_0(x) \equiv 1. \)
2. Low constant conductivity
   \( \lambda_l(x) = \lambda_0 / 2. \)
3. High constant conductivity
   \( \lambda_h(x) = 2\lambda_0. \)
4. Varying conductivity with long period
   \( \lambda_S(x) = 2 \sin(2\pi x / L) \lambda_0. \)
5. Varying conductivity with short period
   \( \lambda_N(x) = 2^{0.5} \sin(9.8\pi x / L) + 0.5 \cos(20.2\pi x / L) \lambda_0. \)
6. Varying conductivity with long + short period
   \( \lambda_{NS}(x) = \lambda_N(x)\lambda_S(x)\lambda_0. \)

Fig. 3 shows the six resulting conductivity profiles. The terms “long” and “short” period refer to the spatial wavelength of the conductivity variation with respect to the total filling length. The open-loop pressure profile that gives the desired front motion for the nominal case is \( P_0(t) = (V^2 / \lambda_0) t, \) where \( V = 0.01, \) and \( \lambda_0 \equiv 1. \) This control is found by setting \( \dot{\lambda}_i = \dot{\lambda}_0, X_i = 0, \) \( t_i = 0, \) and \( \dot{V}_i = V \) in Eq. (13). For all but the nominal case, the resulting front motion will be incorrect. This is demonstrated in Fig. 4, which shows the result of applying the open-loop pressure profile to the six cases. The figure shows the deviation from the desired linear behavior, as given by Eq. (14).

The estimation algorithm, Eq. (10), and control law, Eq. (13), provide much improved tracking of the desired interface motion when \( \lambda \) varies. This is clearly shown in Fig. 5, comparing controller error for the six conductivity profiles. In this example, the control cycle is \( T = 0.5 \) s. Initially, the conductivity estimate is the nominal value, \( \dot{\lambda}_1 = \dot{\lambda}_0 = 1. \) In the constant conductivity cases, the controller converges to the correct \( P_0(t) \) in one time step. For the varying conductivity...
cases, the tracking is not as good, but it is an order of magnitude better than the open-loop case. Fig. 6 shows the corresponding control $P_0(t)$.

The performance of the controller is limited by the control bandwidth, as determined by $\Delta \tau$, where smaller $\Delta \tau$ corresponds to higher bandwidth. This is shown in Fig. 7, the closed-loop error
with $\Delta \tau$ increased to 4 s. Fig. 8 shows the corresponding control pressures. While still significantly better than the open-loop control for the constant conductivity case, the lagging response caused by the long time between updates actually amplifies errors due to the low-frequency sinusoidal conductivity variations in the long period case. The severity of the initial transient errors is also
increased, even for the constant conductivity cases. Clearly, the ability of the controller to accommodate parameter variations is significantly degraded by the time delay and lower bandwidth.

It should be noted that the test cases applied here included conductivity variations of up to a factor of 3. The actual variations due to geometric factors, such as curves in the mold shape, are expected to be lower – on the order of a factor of 1.5 [9].

Fig. 7. Performance of the closed-loop control: $\Delta \tau = 4.0$ s.

Fig. 8. The closed-loop control: $\Delta \tau = 4.0$ s.
5. Possible extension to 2-D

The next challenge for this research program is extending these ideas to 2-D geometries. The plan is to treat the planar case as multiple 1-D flows: each one corresponding to a separate resin injection port. The algorithm developed in the present paper will form the basis of an estimation scheme for the 2-D case, that will allow the conductivity to be mapped out as filling proceeds. Although the details are necessarily speculative at this point, here is a brief sketch of one possible approach.

Consider a 2-D mold, at some instant intermediate in the filling process. If the conductivity of the filled regions is known, then the pressure field and velocity field can be determined by simulation. Assume that is the case. Though the flow is unsteady, and so there are no streamlines, the instantaneous flow field can be integrated as though it were steady. The flow lines originating from each filling port can be traced. In particular, curves can be drawn separating 2-D regions based on which port the flow lines in that region originate. Those curves can then be extended into the unfilled region, either based on geometry, via splines for example, or based on some assumptions concerning the conductivity there. Now let the filling proceed for one estimation interval. By treating the separatrix curves as being fixed, on the assumption that the isobars in the filled region change more slowly than the front advances, the flow associated with each filling port can be treated independently, using the technique developed in this paper, to give the effective conductivity of the region filled by each 1-D “channel”. Thus, at the end of this interval, an effective conductivity estimate can be applied to each filled point. Because the conductivity of every filled region at the start of the process is known (trivially – there are none), this procedure can be applied inductively from the start. When the fill is complete, an effective conductivity can be assigned to the mold according to an irregular grid. This grid may be coarse or fine, depending on the number of fill ports, and the length of the estimation interval.

Such an estimation scheme would be the crudest application of the 1-D method to the 2-D problem. Some obvious possible refinements include iteration to improve estimation accuracy, and applying control based on the estimate of the front shape. Furthermore, some assumptions in the above sketch need to be validated. It is expected that the 1-D scheme will require significant enhancement. Nevertheless, the above serves to indicate the direction of future research, and motivate the usefulness of the 1-D result.

References
