PHOTOTHERMAL DEFLECTION TECHNIQUE

Theory and applications: the experience at “La Sapienza” in Rome

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• PHOTOTHERMAL TECHNIQUES
• PRINCIPLE OF PHOTOTHERMAL DEFLECTION
• THE HEAT DIFFUSION
• MEASUREMENT OF THERMAL DIFFUSIVITY
• OTHER APPLICATIONS

Thanks to

Photothermal at Roma La Sapienza
Grigore L. LEAHU, Stefano PAOLONI,
Concita SIBILIA, Mario BERTOLOTTI
PHOTOTHERMAL EFFECTS
Photothermal reflection signal

\[ S = 2h(x) \cos(\theta) + 2g(x)L \]

- \( S \) displacement
- \( L \) distance between sample and detector
- \( h \) distortion height
- \( g \) distortion gradient
PHOTOTHERMAL TECHNIQUES

Photoreflectance scheme

ΔR/R → Thermal Wave Signal
PHOTOTHERMAL TECHNIQUES
Photoreflectance microscope

Schematic set up

Magnification
PHOTOTHERMAL TECHNIQUES
Radiometric technique

Stephan-Boltzman law
\[ W = \varepsilon \sigma T^4 \leftrightarrow \delta W = 4\varepsilon \sigma T_0^3 \delta T \]
\[ \sigma = 5.67 \times 10^{-12} \text{ Wcm}^{-2}\text{K}^{-4} \]

Lambert law
\[ S = 4\varepsilon \sigma T_0^3 \sin^2(\theta) \delta T \]
\[ S, \text{ PTR signal} \]

Lambert law
\[ S = \varepsilon \sin^2(\theta) T_r(\lambda_d) R(\lambda_d) \frac{\partial W(\lambda_d)}{\partial T} \Delta \lambda \delta T \]

Selective filter at \( \lambda_d \)

\[ W(\lambda_d) = \frac{2\pi h c^2}{\lambda_d^5} \left( \frac{1}{e^{hc/\lambda_d k_B T}} - 1 \right) \]

Tr  transmission of the optics
R  detector sensitivity
Photoacoustic technique

\[
dP = \frac{\gamma P r^2 s_g}{T \left( R^2 l_g + V_r / \pi \right)} dT
\]

**Photoacoustic signal**

\[
s_g = \min \left( l_g, \sqrt{D_g / \pi f} \right)
\]

**Effective cavity length**

...you can hear the light (Bell, 1880)
Photopyroelectric signal

\[ i_p = -pA \frac{d}{dt} \int_0^d T(x,t)dx \]

- \( p \) pyroelectric constant of the material
- \( A \) area of the detector
- \( i_p \) current
- \( T \) temperature rise
PHOTOTHERMAL TECHNIQUES
Other optical techniques

INTERFEROMETRY

THERMAL LENS

DEFLECTION

DIFFRACTION
PHOTOTHERMAL TECHNIQUES
Photothermal deflection technique

Mirage effect

Schematic set up

Trace gas analysis device

Deflection angle

\[ \tilde{\Phi} = \frac{1}{n} \frac{dn}{dT} \int_{\text{path}} \nabla T ds \]
Sunlight reflects off the tree.

Some of the reflected light propagates towards the observer, some towards the ground where it is refracted.

The refracted light results in an inferior mirage, which appears to the observer as a reflection on the ground.
...so objects appear higher.

Light bends due to differing air density...

Magical flying ship

Actual ship
Thermo-optical spectroscopy: Detection by the "mirage effect"

A. C. Boccara, D. Fournier, and J. Badoz
Laboratoire d'Optique Physique, EPCI, ER No 5 du CNRS, 10, rue Vauquelin, 75231 Paris Cedex 05, France
(Received 2 August 1979; accepted for publication 7 November 1979)

A new thermo-optical method based on the sensitive detection of thermal gradients adjacent to heated sample surfaces is described. Room- and low-temperature experiments were performed using this technique, and its advantages over different methods are discussed.

PACS numbers: 78.20.Nv, 07.65. - b, 44.30. + v, 67.40.Pm

When a conventional photoelectric measurement of the absorption coefficient is not feasible, i.e., for weakly absorbing samples and for opaque and diffusing samples, it is possible to use either calorimetric, interferometric, or photoacoustic techniques.\textsuperscript{1,2}

The aim of this letter is to describe a new experimental

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Experimental setup. The sample is illuminated either by a 450-W Xe arc through a J.Y. f/2 monochromator (Figs. 2 and 3) or by a cw dye laser (Fig. 4).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3}
\caption{Thermo-optical absorption spectrum of Nd\textsubscript{3}(MoO\textsubscript{4})\textsubscript{3} crystal. Spectral bandwidth 100 Å, \(\omega/2\pi = 80\) Hz.}
\end{figure}
The amplitude of the light beam deflection at modulation $\omega$ is given by $\phi = (l/n)(dn/dT)(dT/dx)$, where $l$ is the interaction pathway between the probe beam and the temperature gradient. This allows for the measurement of the temperature changes in the sample.

FIG. 2. Thermo-optical absorption spectrum of a Cs$_3$Cr$_3$Cl$_9$ powdered sample. Spectral bandwidth 100 Å, $\omega/2\pi = 300$ Hz.

spectra of a Cs$_3$Cr$_3$Cl$_9$ powdered sample and of a Nd$_2$(MoO$_4$)$_3$ monocystal, respectively.

In order to perform thermo-optical measurements in liquid helium below the $\lambda$ point, we have used the same technique instead of a conventional bolometer. The time-dependent index of refraction gradient is generated by creating a heat standing wave (second sound) in the tail of the helium Dewar ($\sim 10$ mm). Despite the weak $dn/dT$ factor for liquid helium below 2 K, we were able to easily detect absorption

FIG. 4. $^4I_{15/2} \rightarrow ^2H_{11/2}$ transition in Nd$_2$(MoO$_4$)$_3$ at 2 K. $\omega/2\pi = 3450$ Hz. Upper curve, thermo-optical absorption spectrum; lower curve, thermo-optical magnetic circular dichroism spectrum at 0.7 T.
Sensitive photothermal deflection technique for measuring absorption in optically thin media

A. C. Boccara and D. Fournier

Laboratoire d'Optique Physique, E.R. 5 du CNRS, Ecole Superieure de Physique et de Chimie Industrielles, 10, rue Vauquelin, 75231 Paris Cedex 05, France

Warren Jackson and Nabil M. Amer

Applied Physics and Laser Spectroscopy Group, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Received April 14, 1980

A highly sensitive and simple photothermal scheme for determining optical absorptions in condensed-matter samples is presented. $\alpha$ values as low as $10^{-7}$ and $10^{-8}$ were measured for thin films and coatings and for liquids, respectively. A comparison with the thermal lens effect is given, and the experimental factors limiting our sensitivity are discussed.

Fig. 1. Experimental setup. 1, Position sensor; 2, lock-in amplifier; 3, modulator; 4, power meter; $L_1$, 12-cm focal-length lens; $L_2$, 6-cm focal-length lens; $B_1$, beam splitter.

Fig. 2. Thermal-deflection signal versus the separation between the pump and probe beams. The solid lines are the results of the theory given in Ref. 4.
Fig. 3. (A) absorption spectrum of the sixth harmonic of the C—H stretching excitation of neat benzene. \( l = 0.5 \) mm; beam power, 60 mW. (B) the absorption of 0.7% benzene in CCl₄. \( l = 0.5 \) mm; beam power, 60 mW; lock-in-amplifier time constant, 0.3 sec. Bar represents typical errors of \( \pm 2 \times 10^{-6} \) cm⁻¹.

References

5. Silicon Detector Corporation, Newbury Park, California.
Photothermal deflection spectroscopy and detection

W. B. Jackson, N. M. Amer, A. C. Boccara, and D. Fournier

The theory for a sensitive spectroscopy based on the photothermal deflection of a laser beam is developed. We consider cw and pulsed cases of both transverse and collinear photothermal deflection spectroscopy for solids, liquids, gases, and thin films. The predictions of the theory are experimentally verified, its implications for imaging and microscopy are given, and the sources of noise are analyzed. The sensitivity and versatility of photothermal deflection spectroscopy are compared with thermal lensing and photoacoustic spec-

References

Photothermal spectroscopy using optical beam probing: Mirage effect

J. C. Murphy and L. C. Aamodt

Applied Physics Laboratory, The Johns Hopkins University, Laurel, Maryland 20810

(Received 17 January 1980; accepted for publication 2 June 1980)

Optical beam deflection near a heated surface was recently introduced as a method of photothermal spectroscopy. Photothermal spectroscopy (PTS) is closely related to photoacoustic spectroscopy (PAS) in its ability to measure optical properties of opaque solids and liquids. This paper develops a theory of this effect and compares this theory with extensive experimental observations. Both are in excellent agreement. The relationship between this form of PTS and PAS is explicitly developed. Applications to the measurement of thermal diffusivity of gases is described.

PACS numbers: 07.65. — b, 43.35.Sx, 78.20.Hp

in the sample and the justification for the name photoacoustic spectroscopy. The low detection threshold of photoacoustic systems is attributable to the excellent sensitivity of available microphones. Recently Fournier, Boccara, and Bodoz have introduced an optical means for carrying out PAS experiments (“mirage” detection). In this method, temperature-induced changes in the index of refraction of the gas in contact with the sample surface are used to detect light absorption. They report successful measurement of PAS spectra as a function of temperature into the liquid helium range. A significant advantage of this method is that the spatial gradient of the total gas temperature rise associated

PRINCIPLE OF PHOTOTHERMAL DEFLECTION

HELMOLTZ Equation
\[ \nabla^2 V(r) + k^2 n^2(r) V(r) = 0 \]
\[ V(r) = A(r) \cdot e^{i k S(r)} \]

RAY OPTICS Equation
\[ \nabla^2 A + k^2 A \cdot (n^2 - |\nabla S|^2) = 0 \]
\[ A \nabla^2 S + 2 \nabla A \cdot \nabla S = 0 \]

RAY OPTICS Solution
\[ |\nabla S|^2 = n^2 \implies \nabla S = n \cdot \sigma \]

BENDING EFFECT
\[ \frac{dS}{ds} = n \]
\[ \nabla n = \nabla \left( \frac{dS}{ds} \right) \]
\[ \nabla_t n = n \frac{d\sigma}{ds} = \frac{n}{\rho} \tilde{\nu} \]

Light bends only due to the transverse gradient
WEAK DEFLECTION

\[ \nabla_t n = n \frac{d\sigma}{ds} \]

Integration over \( s \)

\[ \sigma(s) - \sigma(0) = \int_0^s \frac{\nabla_t n}{n} \, ds \]

Final ray \( \sigma(s) = \text{sen} \Phi \cdot \vec{v}(0) + \cos \Phi \cdot \sigma(0) \equiv \Phi \cdot \vec{v}(0) + \sigma(0) \)

Deflection formula \( \Phi = \int \frac{\nabla_t n \cdot \vec{v}}{n} \, ds \)

2-D WEAK DEFLECTION

\[ \Phi_{h_1} = \int_{\text{path}} \frac{\nabla_t n \cdot \vec{h}_1}{n} \, ds = \int_{\text{path}} \frac{1}{n} \frac{\partial n}{\partial h_1} \, ds \]

\[ \Phi_{h_2} = \int_{\text{path}} \frac{\nabla_t n \cdot \vec{h}_2}{n} \, ds = \int_{\text{path}} \frac{1}{n} \frac{\partial n}{\partial h_2} \, ds \]
NATURE OF THE DEFLECTION

Taylor expansion of the refractive index

\[ dn = \frac{\partial n}{\partial T} \cdot (T - T_0) + \frac{\partial n}{\partial f_1} \cdot (f_1 - f_{10}) + \ldots + \frac{\partial n}{\partial f_n} \cdot (f_n - f_{n0}) \]

Photothermal deflection

\[
\begin{align*}
\bar{\Phi} &= \Phi_T + \Phi_{f_1} + \ldots + \Phi_{f_n} \\
\Phi &= \int_{\text{path}} \frac{\partial n}{\partial T} \cdot \nabla_i T \, ds + \sum_{i} \int_{\text{path}} \frac{\partial n}{\partial f_i} \cdot \nabla_i f_i \, ds \\
\bar{\Phi} &= \frac{1}{n} \frac{dn}{dT} \int_{\text{path}} \nabla_i T \, ds
\end{align*}
\]
In gases

\[ n(p, T) = 1 + \frac{3Ap}{2RT} \]

<table>
<thead>
<tr>
<th>Tipo di composto</th>
<th>A [m^3 mol^{-1}] *10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aria</td>
<td>4.6</td>
</tr>
<tr>
<td>Ossigeno</td>
<td>4.05</td>
</tr>
<tr>
<td>HCl</td>
<td>6.68</td>
</tr>
<tr>
<td>Vapore acqueo</td>
<td>3.72</td>
</tr>
<tr>
<td>CS_2</td>
<td>22</td>
</tr>
<tr>
<td>C_3H_6O</td>
<td>16.2</td>
</tr>
</tbody>
</table>

\[ \left\{ \begin{array}{l}
\left. \frac{dn}{dT} \right|_{T_o} = -\frac{3Ap}{2RT_o^2} \\
\left. \frac{d^2n}{dT^2} \right|_{T_o} = \frac{3Ap}{RT_o^3}
\end{array} \right. \]

AIR OPTOTHERMAL COEFFICIENT

Best approximation

\[ \frac{dn}{dT} = -7.856 \cdot 10^{-5} \frac{p\left(1 + 3.34 \cdot 10^{-6} p\right)}{T_o^2} \]
$\frac{dn}{dT} = -7.856 \cdot 10^{-5} \left( p \left( 1 + 3.34 \cdot 10^{-6} \frac{p}{T_o^2} \right) \right)$

Air opththermal coefficient

Temperature (K)

Air optothermal coefficient
EXPERIMENTAL RESULTS ON THE AIR OPTOTHERMAL COEFFICIENT

Pressure dependence

Pressure of Nitrogen at 77K

Deflection Signal

Offset (mm)

Maximum deflection signal

Pressure (mBar)
Deflected / Undeflected beam

x=0
distance d

Filter
Position Sensor
Lock-in
PC

SIGNAL DETECTION

DEFLECTED / UNDEFLCTED BEAM

POSITION SENSOR EQUATIONS

Zone A
Zone B
Beam center
Deflected beam
Undeflected beam

Zone C
Zone D

y
z
x

BEAM CENTER
\[ \begin{aligned}
    y_0 &= \Phi_y d \\
    z_0 &= \Phi_z d
\end{aligned} \]

SPOT-SIZE BROADENING
\[ w(d) = w_0 \cdot \sqrt{1 + \left( \frac{\lambda d}{\pi nw_0^2} \right)^2} \approx \frac{\lambda d}{\pi w_0^2} \]

BEAM INTENSITY ON THE DETECTOR
\[ I(y, z) = \frac{2P}{\pi w^2(d)} \cdot e^{-\frac{2(y-y_0)^2+(z-z_0)^2}{w^2(d)}} \]
POWER ON THE DETECTOR

Weak deflection approximation

\[ \text{erf}(z) \equiv \frac{2}{\sqrt{\pi}} z \]

\[ P_A = \frac{P}{4} \cdot \frac{\sqrt{6}}{\sqrt{\pi}} \cdot \left( \frac{z_0 - y_0}{w(d)} \right) = \frac{Pw_0 \sqrt{\pi}}{\lambda \sqrt{2}} \cdot (\Phi_z - \Phi_y) \]

\[ P_B = \frac{P}{4} \cdot \frac{\sqrt{6}}{\sqrt{\pi}} \cdot \left( \frac{z_0 + y_0}{w(d)} \right) = \frac{Pw_0 \sqrt{\pi}}{\lambda \sqrt{2}} \cdot (\Phi_z + \Phi_y) \]

\[ P_C = \frac{P}{4} \cdot \frac{\sqrt{6}}{\sqrt{\pi}} \cdot \left( \frac{y_0 - z_0}{w(d)} \right) = \frac{Pw_0 \sqrt{\pi}}{\lambda \sqrt{2}} \cdot (\Phi_y - \Phi_z) = -P_A \]

\[ P_D = -\frac{P}{4} \cdot \frac{\sqrt{6}}{\sqrt{\pi}} \cdot \left( \frac{z_0 + y_0}{w(d)} \right) = -\frac{Pw_0 \sqrt{\pi}}{\lambda \sqrt{2}} \cdot (\Phi_z + \Phi_y) = -P_B \]

DEFLECTION SIGNAL

**Vertical signal**

\[ \Delta V_v = \frac{P_A + P_B - P_C - P_D}{V_o} \]

\[ \Delta V_l = -\frac{P_A + P_B + P_C - P_D}{V_o} \]

**Lateral signal**

\[ \Delta V_v = \sqrt{8\pi} \frac{w_0}{\lambda} \Phi_z \]

\[ \Delta V_l = \sqrt{8\pi} \frac{w_0}{\lambda} \Phi_y \]
Main applications

• Thermal diffusivity and effusivity measurements
• Absorption spectroscopy
• Effusivity and optical absorption depth profiling
• Measurement of the attenuation in optical waveguides
• Evaluation of the thickness of thin layers
• Trace gas analysis
• Characterization of metallic surfaces
Experimental Setup

- monocromator
- chopper
- Pump lamp
- Probe laser
Photothermal Deflection Spectroscopy Set-up

Monochromator bandwidths

Monochromator

Mirrors

Grating

Chopper

Lens

Deflection angle

Chopper

Sample

Cell

CCl$_4$

x,y,z stage

Sample

Deflection angle

Motor driver

Lock-in Amplifier

PreAmpl

Position sensor

Ref signal

300 W Xe Lamp housing

He-Ne laser

Monochromator

Photothermal Deflection Spectroscopy Set-up

Monochromator bandwidths

1000 nm
700 nm
400 nm
Optical Reflectance Spectroscopy

s-polarized wave

p-polarized wave

Optical Reflectance

θ

Si

AlN

wavelength, nm

Optical reflectance

85°

75°

65°

55°

45°

wavelength, nm

80

60

40

20

0

400

500

600

700

800

80

60

40

20

0

400

5

168x167

s-polarized wave

p-polarized wave

Optical Reflectance Spectroscopy

θ

Si

AlN

wavelength, nm

Optical reflectance

85°

75°

65°

55°

45°

wavelength, nm

80

60

40

20

0

400

500

600

700

800

80

60

40

20

0

400

5

63x211

Courtesy of A.Passaseo
Photothermal Deflection Spectroscopy on AlN

- Calculated optical absorbance 0°
- Optical reflectance 0°
- PDS

Wavelength, nm

Normalized Signal, a.u.
Device under test

**Section**

- QD
- GaAs
- QD
- GaAs
- QD
- GaAs
- QD
- GaAs
- QD
- GaAs
- QD
- GaAs
- Si doped GaAs substrate

- 4 MLs 35 nm
- 4 MLs 35 nm
- 4 MLs 35 nm
- 4 MLs 35 nm
- 4 MLs 5 nm
- 200 nm

Roughness: 3500 nm

Courtesy of A. Passaseo
Photothermal Deflection Spectroscopy

“Back” configuration

Pump beam

Position sensor

GaAs :Si

InGaAs QD

CCl\textsubscript{4} liquid

“Front” configuration

InGaAs QD

GaAs :Si
Experimental Results on Quantum Dots

PTD, QD-InGaAs/GaA-SI

- Phase, deg.
- Wavelength, nm
- QD fronte
- QD retro
- GaAs fronte

PDS, QD-InGaAs/GaA-SI

- Deflection signal, a.u.
- Wavelength, nm
- QD fronte
- QD retro
- GaAs-SI wafer
3D Photonic crystal: SiO$_2$ synthetic opal
Characterization of VO$_2$/SiO$_2$ inverse opals

![Graph showing characterization results with wavelength, nm on the x-axis and deflection signal on the y-axis.]

- Cold phase
- Hot phase

![Diagram showing structural changes between cold and hot phases.]

\[ R_{i,0}, R_{i,1}, \text{Exp} \_11, \text{Exp}_{ph} 11 \]

\[ \lambda_i \]

\[ 500, 550, 600, 650, 700, 750, 800, 850, 900, 950, 1000, 1050, 1100 \]

\[ 10^{-1}, 10^{-2}, 10^{-3} \times 10^3 \]

Wavelength, nm
Main applications

• Thermal diffusivity measurements
• Absorption spectroscopy
• Effusivity and optical absorption depth profiling
• Measurement of the attenuation in optical waveguides
• Evaluation of the thickness of thin layers
• Trace gas analysis
• Characterization of metallic surfaces
THE HEAT DIFFUSION

\[
\begin{align*}
\vec{F} &= -k \nabla T \quad \text{Heat flux} \\
\nabla \cdot \vec{F} + \rho c \frac{\partial T}{\partial t} &= w \quad \text{Energy conservation law} \\
\nabla^2 T - \frac{1}{D} \frac{\partial T}{\partial t} &= -\frac{w}{k} \quad \text{Fourier diffusion equation}
\end{align*}
\]

PULSED REGIME

Green function in pulsed regime
Temperature solution for unitary pulse placed in the origin at \( t=0 \)

\[
G_1(x, y, z, t) = \frac{1}{8\rho c[\pi Dt]^{3/2}} e^{-\left(x^2 + y^2 + z^2\right)/4Dt}
\]
Thermal waves

\[ \frac{\partial^2 T}{\partial x^2} - \frac{1}{D} \frac{\partial T}{\partial t} = -\frac{we^{j\alpha}}{k} \]

\[ \frac{d^2 \tilde{T}}{dx^2} - \beta^2 \tilde{T} = -\tilde{w} \]

\[ \tilde{T}(x, \omega) = Ae^{-\beta x} = Ae^{-(1+j)x/\ell} \]

\[ T(x,t) = Ae^{-x/\ell} \cos(\omega t - x/\ell + \pi/4) \]

Thermal diffusion length

\[ \ell = \sqrt{D/\pi f} \]

Pump beam
THEORY OF THERMAL WAVES

\[ \tilde{T}(z, \omega) = Ae^{-\beta \tilde{r}} = Ae^{-\beta z} = Ae^{-(1+j)z/\ell} \]

\[ T(z, t) = \text{Re} \left[ \tilde{T}(z, \omega)e^{j\omega t} \right] = Ae^{-z/\ell} \cos(\omega t - z/\ell + \varphi) \]

Heating period  Cooling period  Plane source

Complex quantity method
Amplitude of temperature  \( Ae^{-z/\ell} \)
Phase of the temperature  \(-z/\ell\)

\( \ell = \sqrt{D/\pi f} \)

Normalized path  \( z/\ell \)
Amplitude normalized
Phase (degree)
PLANE THERMAL WAVE

- Negative envelope
- Positive envelope

Distance from the source

ωt = 0
ωt = π/3
ωt = π/2
ωt = −π

THE HEAT DIFFUSION
THE HEAT DIFFUSION

THERMAL WAVE GENERATION

1D - PLANE THERMAL WAVE

SAMPLE

SPHERICAL THERMAL WAVE

Pump beam

lens

Surface

laser on hot period

laser off cold period

hot cold

Thermal wavelength
PLANE THERMAL WAVE GENERATION
on the soil due to the seasonal light oscillations

Heat diffusion equation in harmonic regime

\[ \frac{d^2 \tilde{T}}{dz^2} - \left( \frac{1}{D} \frac{d}{dz} \right)^2 \tilde{T} = 0 \]

\[ \beta^2 \]

\[ \tilde{T}(z, \omega) = A \cdot e^{-\beta z} = A \cdot e^{-\frac{j \omega z}{D}} = A \cdot e^{-(1+j)^z/l} \]

Boundary condition on heat flux at the surface

\[ -k \frac{d \tilde{T}}{dz} \bigg|_{z=0} + h \tilde{T} = I \quad \rightarrow \quad (k \cdot \beta + h) \cdot A = I \quad \rightarrow \quad A = \frac{I}{k \beta + h} \]

\[ \tilde{T}(z, \omega) = \frac{I}{k \beta + h} \cdot e^{-\beta z} = \frac{I}{k \sqrt{j \omega D} + h} \cdot e^{-\frac{j \omega z}{D}} \]

\[ T(z, t) = \text{Re} \left[ \tilde{T}(z, \omega)e^{i \omega t} \right] = \frac{I}{e^g \sqrt{\omega}} e^{-z/l} \cos \left( \alpha t - \frac{z}{l} \pi \right) \]
<table>
<thead>
<tr>
<th>ROMA CIAMPINO (1961-1990)</th>
<th>Mesi</th>
<th>Stagioni</th>
<th>Anno</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gen</td>
<td>Feb</td>
<td>Mar</td>
</tr>
<tr>
<td><strong>T. max. media (°C)</strong></td>
<td>11,8</td>
<td>13,0</td>
<td>15,2</td>
</tr>
<tr>
<td><strong>T. min. media (°C)</strong></td>
<td>2,7</td>
<td>3,5</td>
<td>5,0</td>
</tr>
<tr>
<td><strong>T. min. assoluta (°C)</strong></td>
<td>-11,0 (1965)</td>
<td>-6,0 (1965)</td>
<td>-6,5 (1963)</td>
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<tr>
<td><strong>Giorni di calura (T&lt;sub&gt;max&lt;/sub&gt; ≥ 30°C)</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Giorni di pioggia</strong></td>
<td>9</td>
<td>9</td>
<td>9</td>
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<tr>
<td><strong>Umidità relativa media (%)</strong></td>
<td>77</td>
<td>75</td>
<td>72</td>
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<tr>
<td><strong>Eliofania assoluta (ore al giorno)</strong></td>
<td>3,9</td>
<td>4,7</td>
<td>5,4</td>
</tr>
<tr>
<td><strong>Radiazione solare globale media (centesimi di MJ/m²)</strong></td>
<td>652</td>
<td>941</td>
<td>1 396</td>
</tr>
<tr>
<td><strong>Pressione a 0 metri s.l.m. (hPa)</strong></td>
<td>1 017</td>
<td>1 016</td>
<td>1 013</td>
</tr>
<tr>
<td><strong>Vento (direzione-m/s)</strong></td>
<td>NE</td>
<td>4,2</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\frac{\pi}{4} : 2\pi = x : 365d
\]

\[x = 45 \text{ day}\]
Measured temperature oscillations during the year in depth

\[ T(z, t) = \frac{A \cdot I}{e^{\sqrt{\omega}}} e^{-z/\ell} \cos \left( \omega t - \frac{z}{\ell} - \frac{\pi}{4} \right) \]

\[ \omega = \frac{2\pi}{T_{\text{year}}} = \frac{2\pi}{365 \times 86400} \text{ rad/s} \]
Collinear configuration

\[ \Phi = -\left( \frac{dn}{dT} \right) \cdot \frac{P(1 - e^{-\alpha L})}{2 \pi k \ell} \cdot \int_0^\infty \frac{\delta^2 J_1(\delta' \ell) e^{-\frac{1}{2}(\delta^2 + \delta'^2)k^2}}{\delta^2 + 2j} d\delta = -\left( \frac{dn}{dT} \right) \frac{P(1 - e^{-\alpha L})(1 + j)}{2 \pi k \ell} K_1[(1 + j)r/\ell] \]
Phase method

\[ \phi(y) = \varphi_0 - \frac{r}{\mu} \]

\[ \mu = \left| \frac{\Delta r}{\Delta \phi} \right| \]
Transverse configuration

**Surface thermal wave**

\[ T(x,t) = A e^{-x/\ell} \cos(\omega t - x/\ell) \]

- Amplitude
- Phase \( \phi \)

**Thermal diffusion length**

\[ \ell = \frac{\sqrt{D}}{\pi f} \]

**Thermal Diffusivity**

\[ \ell = \frac{\Delta x}{\Delta \phi} \]

Phase (degree)

Horizontal offset (mm)

- 225 Hz
- 400 Hz
- 625 Hz
- 900 Hz
- 1600 Hz

**Surface**

- Pump beam
- Lens
- Sample
- Probe beam

**Vertical deflection**

- Vertical offset \( z \)

**Lateral deflection**

- Lateral offset \( x \)

**Phase offset**

- \( x \)

**Sample**

- Vertical deflection
- Lateral deflection
Transverse configuration

Schema Skimming
Fascio di riscaldamento modulato

\[ \Phi_z = \frac{1}{\pi} \left( \frac{dn}{dT} \right)_o \frac{\alpha P}{k_1} e^{\frac{j\omega^2}{8D_o}} \int_0^\infty \frac{\sqrt{\delta^2 + 2jD_1/D_o} e^{-\left(\frac{\delta^a}{\ell}\right)^2/8z\sqrt{\delta^2 + 2jD_1/D_o}}}{\left(\sqrt{\delta^2 + 2j + \alpha \ell}\right)\left(\sqrt{\delta^2 + 2j + k_o/k_1}\right)} \cos \left(\frac{\delta y}{\ell}\right) d\delta \]

\[ \Phi_y = -\frac{1}{\pi} \left( \frac{dn}{dT} \right)_o \frac{\alpha P}{k_1} e^{\frac{j\omega^2}{8D_o}} \int_0^\infty \frac{\sqrt{\delta^2 + 2j + \alpha \ell}\left(\sqrt{\delta^2 + 2j + k_o/k_1}\right)}{\left(\sqrt{\delta^2 + 2j + \alpha \ell}\right)\left(\sqrt{\delta^2 + 2j + k_o/k_1}\right)} \sin \left(\frac{\delta y}{\ell}\right) d\delta \]
\[ \Phi_y = -\frac{1}{\pi} \left( \frac{dn}{dT} \right)_o \frac{\alpha P}{k_1} \int_0^\infty \frac{\delta \sin \left( \frac{\delta}{k_1} \right) d\delta}{\sqrt{\delta^2 + 2 j + \alpha \ell}} \left\{ \alpha \ell \to 0 \right\} \left( \frac{dn}{dT} \right)_o \frac{\alpha P e^{-\left(1 + j\right)\frac{y}{\ell}}}{k_1} \right. \\
\left. \alpha \ell \to \infty \left( \frac{dn}{dT} \right)_o \frac{-(1 + j)P}{\pi k_1 \ell} K_1 \left[ (1 + j)\frac{y}{\ell} \right] \right. \\
\varphi(y) = \varphi_o - \frac{y}{\mu} \quad \mu = \left| \frac{\Delta y}{\Delta \phi} \right| \\
\text{Fase deflessione laterale (radianti)} \\
\text{Distanza normalizzata} \]
On the photodeflection method applied to low thermal diffusivity measurements

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(Received 18 August 1992; accepted for publication 24 February 1993)

The photodeflection method when applied to measure the low thermal diffusivity of some materials gives inconsistent results. In this article a way to extend the thermal diffusivity range of measurements using the phase of the photodeflection signal is presented. A comparison with computer simulations and experimental results shows good agreement.

\[ l_c = \sqrt{\frac{D}{\pi \nu}} + b \cdot z \]

FIG. 4. Equimodulus (a) and equiphase (b) thermal surfaces for a diffusivity ratio \( D_1/D_0 = 0.25 \) and \( a/I_1 = 0.2 \), as a function of the \( r/I_1 \) (abscissa) and \( z/I_1 \) (ordinate), respectively. The phase shift between two contiguous surfaces is \( \pi/10 \).

FIG. 16. Characteristic length as a function of \( 1/\nu \). Comparison between experimental data and numerical analysis: (\( \times \)) exp. \( z = 0 \) \( \mu \)m, (\( \Delta \)) exp. \( z = 50 \) \( \mu \)m, (---) theory. Pump power = 15 mW, chopper frequency = 9 Hz, \( D_1 = 0.005 \) cm\(^2\)/s.
Analysis of the photothermal deflection technique in the surface reflection scheme: Theory and experiment

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(Received 18 June 1997; accepted for publication 3 October 1997)

The photothermal deflection technique has been usually applied, for the thermal diffusivity measurements, in the transverse skimming scheme. To overcome some limitations of the skimming, a surface reflection scheme (i.e., bouncing scheme) has been introduced in which the probe beam is reflected from the sample surface. In this configuration the probe beam deflection is obtained as a result of two different mechanisms: the thermal gradient in the gas near to the heated sample (mirage) and the sample surface deformation due to the thermal expansion (displacement). The superposition of these two effects must be taken into account when deriving the thermal diffusivity. In this article the mirage and the displacement have been studied from a theoretical and experimental point of view, and a new method for the measurement of thermal diffusivity in the bouncing scheme is presented. A special setup is described to obtain separately the mirage and the displacement signals from which the thermal diffusivity and the thermal expansion coefficient can be derived. The experimental values for different samples obtained by applying our method are in agreement with the literature values. © 1998 American Institute of Physics.


TABLE I. Thermal diffusivity end expansion for several materials.

<table>
<thead>
<tr>
<th>Sample Type of material (1)</th>
<th>Thermal diffusivity (cm²/s)</th>
<th>Expansion (K⁻¹)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample measured in skimming scheme</td>
<td>Measured in bouncing scheme</td>
<td></td>
</tr>
<tr>
<td>InP</td>
<td>0.44</td>
<td>0.44 ± 0.02</td>
</tr>
<tr>
<td>silicon</td>
<td>0.89</td>
<td>0.95 ± 0.07</td>
</tr>
<tr>
<td>As₂S₃</td>
<td>0.003</td>
<td>0.003 ± 0.0001</td>
</tr>
<tr>
<td>glass</td>
<td>0.0077 ± 10%</td>
<td>0.008 ± 10%</td>
</tr>
</tbody>
</table>

See Ref. 27.
bouncing in vacuum
pure mirage
New photothermal deflection method for thermal diffusivity measurement of semiconductor wafers

M. Bertolotti, V. Dorogan, G. Liakhov, R. Li Voli, S. Paoloni, and C. Sibilia
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(Received 10 September 1996; accepted for publication 3 December 1996)

The photothermal deflection technique is applied in transverse configuration to measure the thermal diffusivity of semiconductor wafers. The large size of these samples inhibits the possibility to make the probe beam skim the sample at a small height which is required for a direct thermal diffusivity measurement. To overcome this problem, three new experimental schemes are proposed, each one based on a different geometry of the heat diffusion (one-, two-, or three-dimensional scheme). In particular for the 3D experimental scheme, a new mirage setup is described which uses two crystalline prisms 6 mm apart from each other to let the probe beam skim 50±3 μm high over the sample surface, with a spot size of 22 μm. The main advantages of this setup, here discussed, are the obtained low probe beam height which is, moreover, independent of the sample dimensions, and the cheap technology to produce the necessary high-quality prisms. The performances of the new schemes have been tested by comparing, for well-known semiconductor wafers (InSb, InAs, InP, GaAs, GaP, Ge, and Si), the experimentally measured thermal diffusivity with the values reported in the literature. © 1997 American Institute of Physics. [S0034-6748(97)03703-9]

<table>
<thead>
<tr>
<th>Sample</th>
<th>Type</th>
<th>( L ) (μm)</th>
<th>( D_\text{S} ) (cm(^2)/s) (Fig. 1)</th>
<th>( D_\text{G} ) (cm(^2)/s) (Fig. 2)</th>
<th>( D_\text{B} ) (cm(^2)/s) (Fig. 3)</th>
<th>Nominal value (cm(^2)/s)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>InSb</td>
<td>n</td>
<td>350</td>
<td>0.20</td>
<td>0.21</td>
<td>0.19</td>
<td>0.19</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>350</td>
<td>0.20</td>
<td>0.21</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>InAs</td>
<td>n</td>
<td>1300</td>
<td>0.21</td>
<td>0.20</td>
<td>0.21</td>
<td>0.19</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>350</td>
<td>0.20</td>
<td>0.21</td>
<td>0.19</td>
<td>0.19</td>
<td>2</td>
</tr>
<tr>
<td>InP</td>
<td>i</td>
<td>330</td>
<td>0.40</td>
<td>0.45</td>
<td>0.44</td>
<td>0.46</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>400</td>
<td>0.39</td>
<td>0.43</td>
<td>0.42</td>
<td>0.45</td>
<td>3</td>
</tr>
<tr>
<td>GaAs</td>
<td>i</td>
<td>350</td>
<td>0.25</td>
<td>0.26</td>
<td>0.25</td>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>350</td>
<td>0.28</td>
<td>0.27</td>
<td>0.28</td>
<td>0.26</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>350</td>
<td>0.23</td>
<td>0.24</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GaP</td>
<td>i</td>
<td>380</td>
<td>0.43</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>400</td>
<td>0.36</td>
<td>0.38</td>
<td>0.38</td>
<td>0.37</td>
<td>12</td>
</tr>
<tr>
<td>Ge</td>
<td>n</td>
<td>470</td>
<td>0.76</td>
<td>0.83</td>
<td>0.80</td>
<td>0.88</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>250</td>
<td>0.87</td>
<td>0.66</td>
<td>0.64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A cryostatic setup for the low-temperature measurement of thermal diffusivity with the photothermal method

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(Received 5 June 1995; accepted for publication 5 July 1995)

A cryostatic setup is described to perform photothermal deflection measurements from room temperature to 77 K. The setup uses gaseous nitrogen as a medium where the photodeflection is produced. The ability of the system to work is demonstrated presenting some measurements of thermal diffusivity of high-temperature superconductor samples and of yttrium-iron garnets with variable aluminum content. © 1995 American Institute of Physics.

\[
\text{Thermal Diffusivity (cm}^2/\text{s)} \]

\[
\begin{array}{cccc}
80 & 90 & 100 \\
0.05 & 0.06 & 0.07 \\
\text{SBPCON} & \text{NORMAL STATE} & \text{YBCO} \\
f = 16 \text{ Hz} \\
\end{array}
\]

Collaboration  
CSM, Alenia, IRTEC CNR, Univ. Trieste
New method for the study of mirror heating of a semiconductor laser diode and for the determination of thermal diffusivity of the entire structure

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(Received 20 July 1994; accepted for publication 24 July 1994)

A new method based on the photothermal deflection technique is described to determine the mirror temperature of a semiconductor laser diode as a function of intensity of drive current. The device’s effective thermal diffusivity can also be measured. A short theoretical discussion is presented together with experimental measurements performed on three different kinds of laser diodes. © 1994 American Institute of Physics.

Es. LASER DIODE DOUBLE HETEROSTRUCTURE
AlGaAs/GaAs, λ=800 nm

COPPER HEAT SINK
medium 2

ACTIVE REGION
BULK medium 1

AIR medium 3

PROBEBEAM

TEMPERATURE DIODE N°2 (K)

SEMICONDUCTOR MONOCRYSTAL
Drive Current 70mA
Threshold Current 50mA

COPPER HEAT SINK

offset from the mirror (µm)

225 Hz
2.5 KHz
10 KHz

current, mA

|δΦ| (rad×10^6)

|Φ| (rad×10^6)

0 20 40 60 80 100

0 20 40 60

h = 0.120 mm
W = 0.420 mm
L = 0.200 mm
d = 0.008 mm
a = 0.010 mm
Characterization of SiO$_2$ /GaN opal sample

<table>
<thead>
<tr>
<th>Sample</th>
<th>n.1</th>
<th>n.2</th>
<th>n.3</th>
<th>n.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composition</td>
<td>SiO$_2$ unfilled</td>
<td>SiO$_2$/GaN 25%</td>
<td>SiO$_2$/GaN 70%</td>
<td>SiO$_2$/GaN 70% inverted</td>
</tr>
<tr>
<td>Thermal diffusivity, cm$^2$/s</td>
<td>$1.4 \times 10^3$</td>
<td>$1.6 \times 10^3$</td>
<td>$3.2 \times 10^3$</td>
<td>$0.62 \times 10^3$</td>
</tr>
</tbody>
</table>

Characteristic length, $\mu$m

- Unfilled
- Silica
- GaN 25%
- GaN 70%
- GaN inverted
THERMAL WAVE REFLECTION AND REFRACTION

for plane waves

\[
\begin{align*}
\tilde{T}_1(x, z) &= Ae^{-\beta_1 [\sin(\theta_1) x + \cos(\theta_1) z]} + rAe^{-\beta_1 [\sin(\theta_1') x - \cos(\theta_1') z]}, \\
\tilde{T}_2(x, z) &= tAe^{-\beta_2 [\sin(\theta_2) x + \cos(\theta_2) z]}
\end{align*}
\]

Medium 1  \hspace{1cm} \text{Medium 2}

\[
\begin{align*}
\tilde{T}_1 &= \tilde{T}_2 & \text{Temperature must be conserved at } z=0 \\
\frac{\partial \tilde{T}_1}{\partial z} &= \frac{\partial \tilde{T}_2}{\partial z} & \text{Normal heat flux must be conserved at } z=0
\end{align*}
\]
Thermal wave reflection and refraction: Theoretical and experimental evidence

M. Bertolotti, a) G. L. Liakhov, b) R. Li Voti, S. Paolini, and C. Sibilia
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(Received 7 July 1998; accepted for publication 23 December 1998)

This article describes and proves the basic phenomena which take place when thermal waves approach an interface between two media: the reflection and the refraction. In synthesis the Snell law for plane thermal waves is proved, both theoretically and experimentally, by means of the mirage technique. © 1999 American Institute of Physics. [S0021-8979(99)02307-5]
“Thermal Snell law”

\[
\begin{align*}
\theta_1' &= \theta_1 \\
\frac{1}{\sqrt{D_1}} \sin(\theta_1) &= \frac{1}{\sqrt{D_2}} \sin(\theta_2) \\
\theta_1 &\leq \theta_{\text{lim}} = \arcsin(\sqrt{D_1/D_2})
\end{align*}
\]

“Thermal Fresnel coefficients”

\[
\begin{align*}
r &= \frac{e_1 \cos(\theta_1) - e_2 \cos(\theta_2)}{e_1 \cos(\theta_1) + e_2 \cos(\theta_2)} \\
t &= \frac{2e_1 \cos(\theta_1)}{e_1 \cos(\theta_1) + e_2 \cos(\theta_2)}
\end{align*}
\]

What the thermal effusivity is?

\[
e = k/\sqrt{D} = \sqrt{k\rho c}
\]

Numerical simulation of the temperature field at the Invar-Air interface. The diffusivities are \(D_{\text{Invar}} = 0.05 \text{ cm}^2/\text{s},\) \(D_{\text{air}} = 0.2 \text{ cm}^2/\text{s}.\) The incidence angle is \(\theta_1 = 20^\circ < \theta_{\text{lim}} = 30^\circ\) and the refracted angle is consequently \(\theta_2 = 43^\circ.\)
THERMAL WAVE REFLECTION

experimental evidence

**Experimental setup**

- Thermal mirror
- Incident wave
- Reflected wave
- Absorbing layer
- Pump beam
- Probe

**Temperature in air**

\[ T_{\text{air}}(x, z) = Ae^{-\beta_{\text{air}}z} + rAe^{\beta_{\text{air}}[\cos(2\theta)z - \sin(2\theta)x]} \]

**Deflection in air**

\[ \Phi_x = \frac{1}{n} \left( \frac{dn}{dT} \right) \int_y \frac{\partial T_{\text{air}}}{\partial x} dy = \frac{1}{n} \left( \frac{dn}{dT} \right) L_{\text{eff}} \frac{\partial T_{\text{air}}}{\partial x} \]

\[ \Phi_z = \frac{1}{n} \left( \frac{dn}{dT} \right) \int_y \frac{\partial T_{\text{air}}}{\partial z} dy = \frac{1}{n} \left( \frac{dn}{dT} \right) L_{\text{eff}} \frac{\partial T_{\text{air}}}{\partial z} \]

**Factor independent on the angle**

\[ C = -\frac{1}{n} \left( \frac{dn}{dT} \right) L_{\text{eff}} \beta_{\text{air}} A \]

**Final expression for the deflection**

\[ \tilde{\Phi}_x = C \left[ rsin(2\theta)e^{\beta_{\text{air}}[\cos(2\theta)z - \sin(2\theta)x]} \right] \equiv C \left[ rsin(2\theta) \right] \]

\[ \tilde{\Phi}_z = C \left[ e^{-\beta_{\text{air}}z} - r \cos(2\theta)e^{\beta_{\text{air}}[\cos(2\theta)z - \sin(2\theta)x]} \right] \equiv C \left[ 1 - r \cos(2\theta) \right] \]
THERMAL WAVE REFLECTION

experimental evidence

\[ \tilde{\Phi}_x = C \left[ r \sin(2\theta)e^{\beta_{air}[\cos(2\theta)z - \sin(2\theta)x]} \right] \approx C \left[ r \sin(2\theta) \right] \]

\[ \tilde{\Phi}_z = C \left[ e^{-\beta_{air}z} - r \cos(2\theta)e^{\beta_{air}[\cos(2\theta)z - \sin(2\theta)x]} \right] \approx C \left[ 1 - r \cos(2\theta) \right] \]
THERMAL WAVE REFLECTION
experimental evidence

**Deflection ratio**

\[ R = \frac{\tilde{\Phi}_x}{\tilde{\Phi}_z} = \frac{r \sin(2\theta)}{1 - r \cos(2\theta)} \approx \frac{-\sin(2\theta)}{1 + \cos(2\theta)} = -\tan(\theta) \]
THERMAL WAVE REFRACTION

experimental evidence

Experimental setup

Reflected thermal wave (in air)
\[ \tilde{T}_2(x, z) = t A e^{\beta_2 z} = t A e^{-\beta_2 [\sin(\theta_2 - \theta_1)x + \cos(\theta_2 - \theta_1)z]} \]

Deflection in the second medium (air)
\[
\begin{align*}
\tilde{\Phi}_x &= C \left[ t \sin(\theta_2 - \theta_1) e^{-\beta_2 [\sin(\theta_2 - \theta_1)x + \cos(\theta_2 - \theta_1)z]} \right] \\
\tilde{\Phi}_z &= C \left[ t \cos(\theta_2 - \theta_1) e^{-\beta_2 [\sin(\theta_2 - \theta_1)x + \cos(\theta_2 - \theta_1)z]} \right]
\end{align*}
\]

Deflection ratio
\[ R = \frac{\tilde{\Phi}_x}{\tilde{\Phi}_z} = \tan(\theta_2 - \theta_1) \]
THERMAL WAVE REFRACTION
experimental evidence
ANOMALOUS THERMAL REFRACTED FIELD

What happens when the Snell law is not valid? \( \theta_1 > \theta_{\text{lim}} = \arcsin\left(\sqrt{D_1/D_2}\right) \)

\[
\tilde{T}_{2\pm}(\rho, \zeta) = Be^{-\left(\frac{1+j}{\ell_1}\right)\rho} \pm \left(\frac{1-j}{\ell_2}\right)\zeta \sqrt{\frac{D_2}{D_1}} \sin^2(\theta_1) - 1
\]

Temperature field at the Invar- Air interface. The incidence angle is \( \theta_1 = 70^\circ > \theta_{\text{lim}} \).
THERMAL WAVE REFRACTION

experimental evidence

\[ R = \frac{\Phi_x}{\Phi_z} = \tan(\theta_2 - \theta_1) \quad \Rightarrow \quad \theta_2 = \theta_1 + \arctg[R(\theta_1)] \]

\[ \sin \theta = \frac{D_{\text{InP}}}{D_{\text{air}}} \]

**Snell law**

\[ \frac{l}{\sqrt{D_1}} \sin(\theta_I) = \frac{l}{\sqrt{D_2}} \sin(\theta_2) \]
THERMAL WAVE INTERFEROMETRY

BASIC PRINCIPLE

To generate plane thermal waves of a given frequency at the front surface of the sample by heating it periodically with a pump laser beam. The waves propagate inside the structure and, if they approach a buried layer with different thermal properties, they are partially reflected giving rise, together with the incident waves, to an interference effect at the front surface.

APPLICATIONS

Nondestructive evaluation of the thermophysical properties and the thickness of layered samples

DETECTION

Photoacoustic
Radiometry
Photothermal Deflection techniques
THERMAL WAVE INTERFEROMETRY

INTERNAL TEMPERATURE RISE

\[ T(z) = Ae^{-\beta z} + Be^{\beta z} \]

MATERIAL/BULK INTERFACE

\[ R = \frac{e_m - e_{\text{bulk}}}{e_m + e_{\text{bulk}}} \]

\[ R = \frac{\text{reflected wave}}{\text{incidence wave}} = \frac{Be^{\beta L}}{Ae^{-\beta L}} \]

\[ B = R \cdot A \cdot e^{-2\beta L} \]

BOUNDARY CONDITIONS AT THE SURFACE (z=0)

\[ T(0) = A + B \]

\[ I = -k \frac{dT}{dz} = k\beta A(1 - e^{-2\beta L}) \]

\[ A = \frac{I}{k\beta (1 - \text{Re}^{-2\beta L})} \]

Temperature

heat flux

I; Power intensity
THERMAL WAVE INTERFEROMETRY

TEMPERATURE RISE AT THE SURFACE

\[ T_{air}(\phi) = \frac{I}{k\beta} \left( \frac{1 + R e^{-2(1+j)L\sqrt{\pi f / D}}}{1 - \text{Re}^{-2(1+j)L\sqrt{\pi f / D}}} \right) \]

\[ R = \frac{e_m - e_{bulk}}{e_m + e_{bulk}} \]

PHASE SIGNAL

\[ \varphi(\sqrt{f}) = -\arctan \left( \frac{2R\sin(2L\sqrt{\pi f / D})e^{-2L\sqrt{\pi f / D}}}{1 - R^2 e^{-4L\sqrt{\pi f / D}}} \right) \]
TEMPERATURE RISE AT THE SURFACE

PHASE SIGNAL

Note that the interference effect (oscillation) is seen when the cavity length is of the same order of the thermal diffusion length of the resonator and when the coefficient $R$ is large enough.

The maximum oscillation is of $45^\circ$ degrees and is obtained in case of perfect reflection from the buried layer with $R=\pm 1$. 

$$T_{\text{air}}(o) = \frac{I}{k\beta} \left( 1 + R e^{-2(1+j)\sqrt{\pi f} / D} \over 1 - \text{Re}^{-2(1+j)\sqrt{\pi f} / D} \right)$$

$$\varphi(\sqrt{f}) = -\arctan \left( \frac{2R\sin(2L\sqrt{\pi f / D}) e^{-2L\sqrt{\pi f / D}}}{1 - R^2 e^{-4L\sqrt{\pi f / D}}} \right)$$
THERMAL WAVE INTERFEROMETRY

Diffusivity measurement by Mirage

\[ \hat{T}_{\text{surf}} = \frac{I}{(e_c + e_{\text{air}})\sqrt{j\omega}} \cdot \frac{1 + R_2 \exp[-2(1 + j)L/\ell_c]}{1 - R_1 R_2 \exp[-2(1 + j)L/\ell_c]} \]

\[ \Delta \phi = \phi - \phi_{\text{ref}} = -\arctan \left( \frac{2R_2 \cdot \sin(2L/\ell_c) \cdot e^{-2L/\ell_c}}{1 - R_2^2 \cdot e^{-4L/\ell_c}} \right) \]

\[ R_1 = \frac{e_c - e_{\text{air}}}{e_c + e_{\text{air}}} \approx 1 \quad R_2 = \frac{e_c - e_b}{e_c + e_b} \]

Inox \( L = 200 \mu m \)

\( D = 0.04, \ 0.046 \text{ or } 0.06 \text{ cm}^2/\text{s} \)
**THERMAL WAVE INTERFEROMETRY**

*Diffusivity measurement by Mirage*

\[ \hat{T}_{\text{surf}} = \frac{I}{(e_c + e_{\text{air}})\sqrt{j\omega}} \cdot \left[ \frac{1 + R_2 \exp\left[-2(1 + j)L/\ell_c\right]}{1 - R_1 R_2 \exp\left[-2(1 + j)L/\ell_c\right]} \right] \]

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*Inox L=200µm*

*D=0.046 cm²/s*

*Spot size = 1mm, 2.3mm, 5mm*
**THERMAL WAVE INTERFEROMETRY**

*Diffusivity measurement by Mirage*

\[ \hat{T}_{surf} = \frac{I}{(e_c + e_{air})\sqrt{j\omega}} \left[ \frac{1 + R_2 \exp[-2(1 + j)L/l_c]}{1 - R_1R_2 \exp[-2(1 + j)L/l_c]} \right] \]

\[ \Delta\varphi = \varphi - \varphi_{ref} = -\text{arc tan} \left( \frac{2R_2 \cdot \sin(2L/l_c) \cdot e^{-2L/l_c}}{1 - R_2^2 \cdot e^{-4L/l_c}} \right) \]

\[ R_1 = \frac{e_c - e_{air}}{e_c + e_{air}} \approx 1 \quad R_2 = \frac{e_c - e_b}{e_c + e_b} \]

**Inox L=200µm**

**D=0.046 cm²/s**

*Spot size = 2.3 mm*

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**Graph:**
- X-axis: Thermal diffusivity, cm²/s
- Data points: (1), (2)

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**Diagram:**
- Pump beam
- Vertical Deflection
- Illuminated area
- Air
- Probe beam
- Opaque coating
- Bulk
THERMAL WAVE INTERFEROMETRY

Diffusivity measurement by Mirage – Reflectivity

\[
\Gamma_{surf}(f) = \frac{\sqrt{f \cdot \hat{T}_{surf}(f)} - \lim_{p \to \infty} \sqrt{p \cdot \hat{T}_{surf}(p)}}{\sqrt{f \cdot \hat{T}_{surf}(f)} + \lim_{p \to \infty} \sqrt{p \cdot \hat{T}_{surf}(p)}} = R_2 \cdot \exp \left[-2(1 + j)\frac{L\sqrt{\pi f}}{\sqrt{D_c}} \right] \implies R_2 = \frac{e_c - e_b}{e_c + e_b}
\]

\[
\begin{align*}
\ln(|\Gamma_{surf}|) & = -2L\sqrt{\pi f}/\sqrt{D_c} + \ln[R_2] \\
\arg(\Gamma_{surf}) & = -2L\sqrt{\pi f}/\sqrt{D_c}
\end{align*}
\]

Inox \( L=200\,\mu m \)

D=0.046 cm\(^2\)/s

Spot size = 2.3 mm

Reflectivity vs Frequency square root, Hz\(^{1/2}\)
Main applications

- Thermal diffusivity and effusivity measurements
- Absorption spectroscopy
- Effusivity and optical absorption depth profiling
- Measurement of the attenuation in optical waveguides
- Evaluation of the thickness of thin layers
- Trace gas analysis
- Evaluation of the photoelastic constants
IR PDS device for trace gas analysis

- Ge, Brewster angle
- Gas chamber
- He-Ne probe
- CO₂ pump
- Position Sensor

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Collinear configuration

Trace Gas Analysis – Infrared Photothermal Deflection Spectroscopy

Pump CO$_2$

Probe He-Ne

Cold zone

Hot zone

Cold zone

Deflected beam

Undeflected beam

Collinear configuration

Photothermal deflection angle

\[ \Phi(y) = -2 \left( \frac{1}{n} \frac{dn}{dT} \right) \frac{P(1 - e^{-\alpha L})}{\alpha \rho c \pi^2 w^2} \left( \frac{y}{w} \right) \cdot e^{-\left(\frac{y}{w}\right)^2} \equiv A \alpha L \]
**Experimental results – Test on CO₂**

Spettro di assorbimento CO₂
**Experimental Results – Test on C$_2$H$_4$**

Spettro di assorbimento di una miscella 50 ppm C$_2$H$_4$.

$10P(14)$ di C$_2$H$_4$
Experimental results on carbossimetilcellulosa

<table>
<thead>
<tr>
<th>Temperature of the treatment</th>
<th>Concentration of the emitted ethylene</th>
</tr>
</thead>
<tbody>
<tr>
<td>450°C</td>
<td>7.07 ppm</td>
</tr>
<tr>
<td>480°C</td>
<td>46.8 ppm</td>
</tr>
</tbody>
</table>
Experimental setup

Pump laser beam
1 D illumination

Absorbing layer
1° thermal mirror (glass)

L (Cavity Length)

Forward wave
Backward wave

Probe beam

2° thermal mirror
THERMAL WAVE RESONATOR

Experimental evidence in air

Photothermal deflection signal

- Amplitude logarithm
- Phase

Cavity length (mm)

frequency 36 Hz
CONCLUSIONS

- PHOTOTHERMAL TECHNIQUES
- PRINCIPLE OF PHOTOTHERMAL DEFLECTION
- THE HEAT DIFFUSION
- MEASUREMENT OF THERMAL DIFFUSIVITY
- OTHER APPLICATIONS