Inverse problems, regularization, and applications to thermal signal reconstruction

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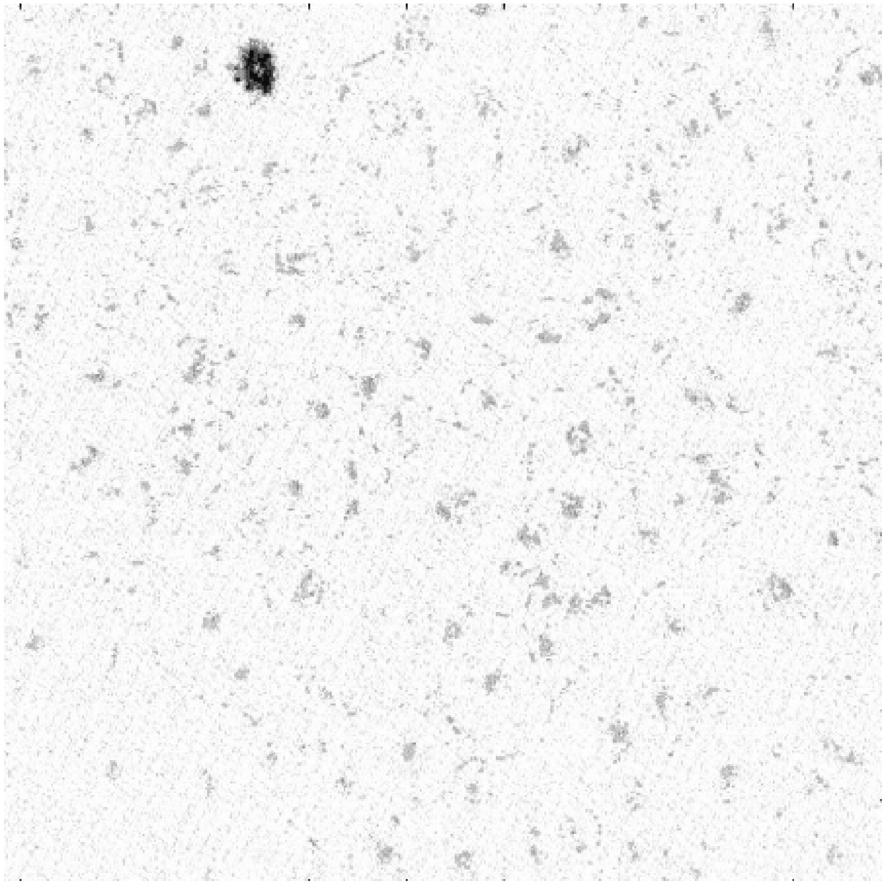
Our Goal is: to look below the surface



- Remote nondestructive optical imaging: OCT, THz, Photoacoustics, Laser – Ultrasound, Thermography
- Spatial resolution in in the micron and mm-range
- „additional information“: e.g.. Optical and acoustical anisotropy; detection of residual stress/strain, adhesion of interfaces

Image resolution and information

6KByte

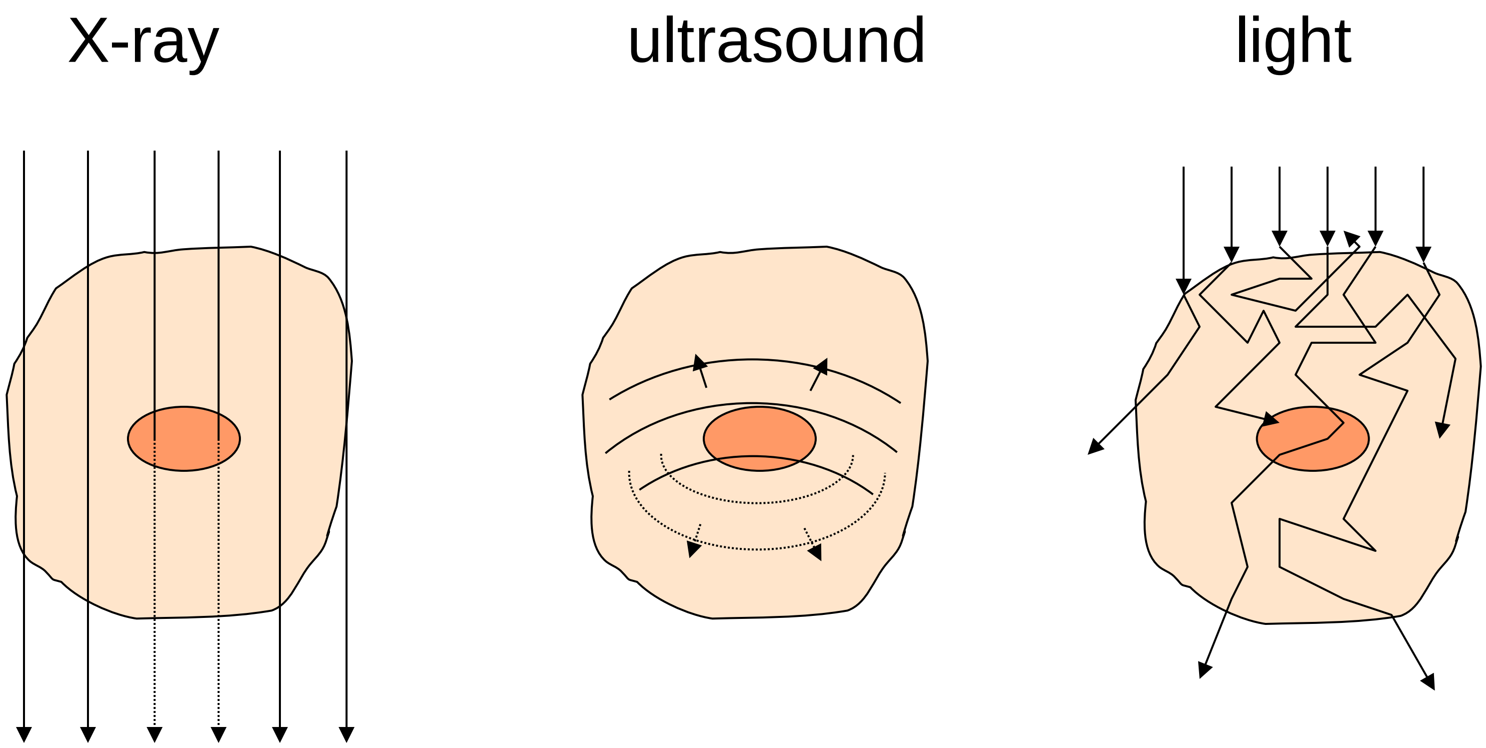
34KByte

280KByte

Outline

- Motivation: Spatial resolution and information content
- High significance of fluctuations for non-destructive imaging (inverse problem)
    - Thermography or photoacoustic imaging as an example.
    - Temperature or pressure are time dependent random variables: mean values are described by differential equations, e.g. diffusion or wave equations.
    - Macroscopic systems: fluctuations are small (thermodynamic limit), but for inverse problems they get important, because they are highly amplified.
- Stochastic thermodynamics of a "kicked" process
    - Good news: information loss (= loss in spatial resolution) due to fluctuations is equal to the mean entropy production (= dissipated heat divided by temperature) → Macroscopic mean value equations describe also influence of fluctuations
- Physical background of regularization: cut-off e.g. for truncated SVD if no additional information goes along with those eigenvalues.
- Resolution limits for pulse thermography and thermographic psf

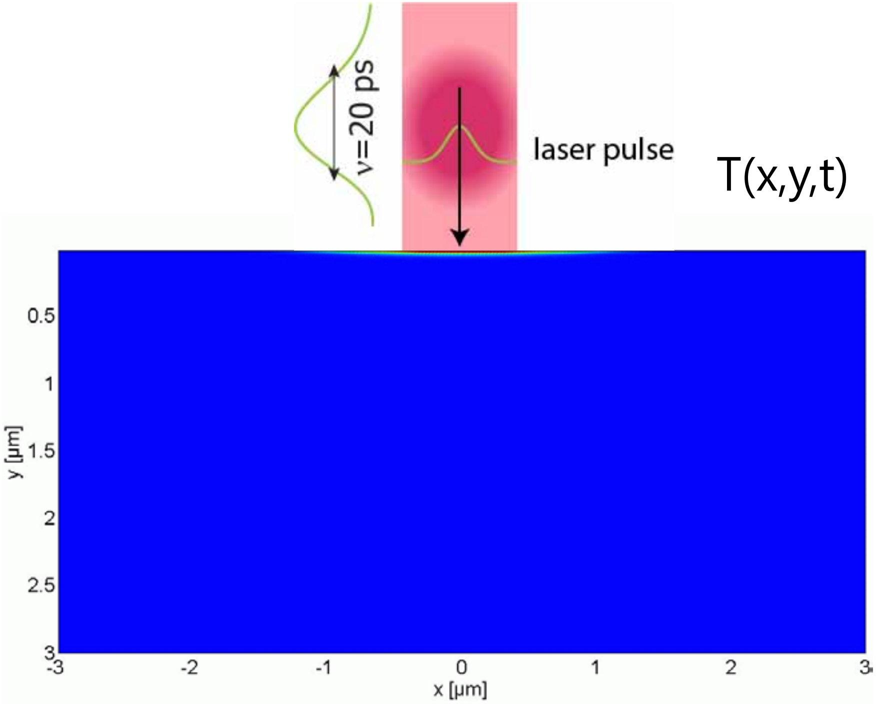
Imaging techniques



Laser-generated wave fields

Process of laser-heating:

- Absorption of light
- Heating and thermal diffusion



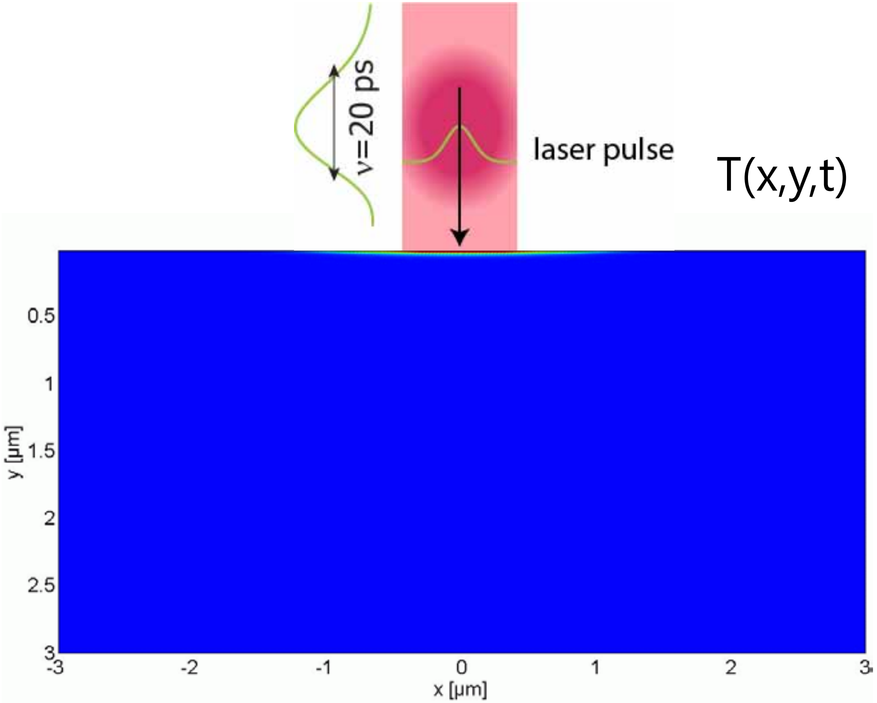
Laser-generated wave fields

Process of laser-heating:

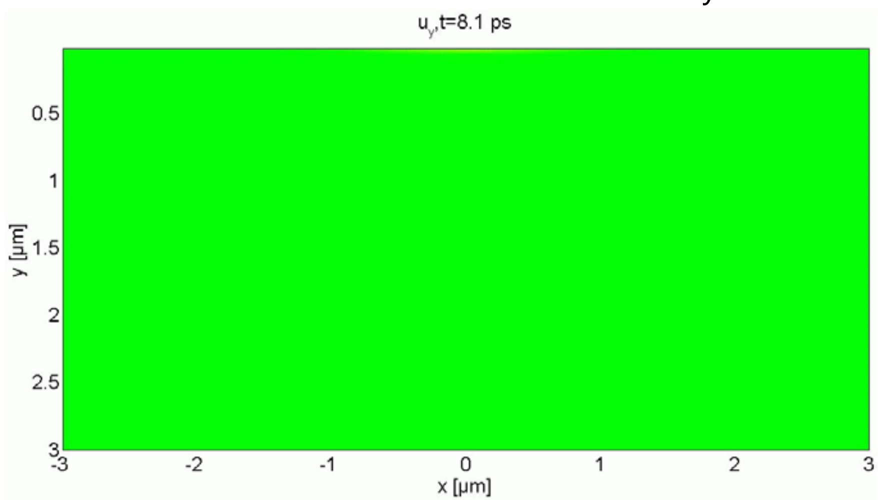
- Absorption of light
- Heating and thermal diffusion

Response of the continuum:

- thermal expansion
- generation of elastic waves

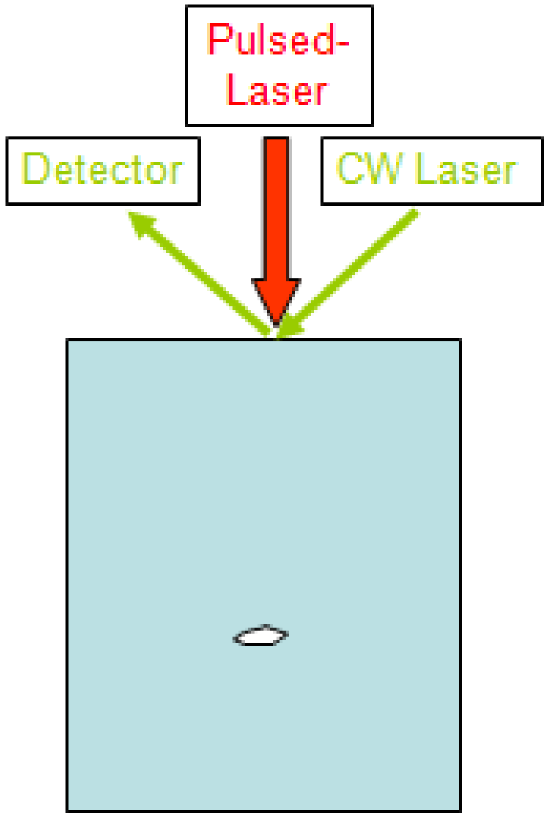


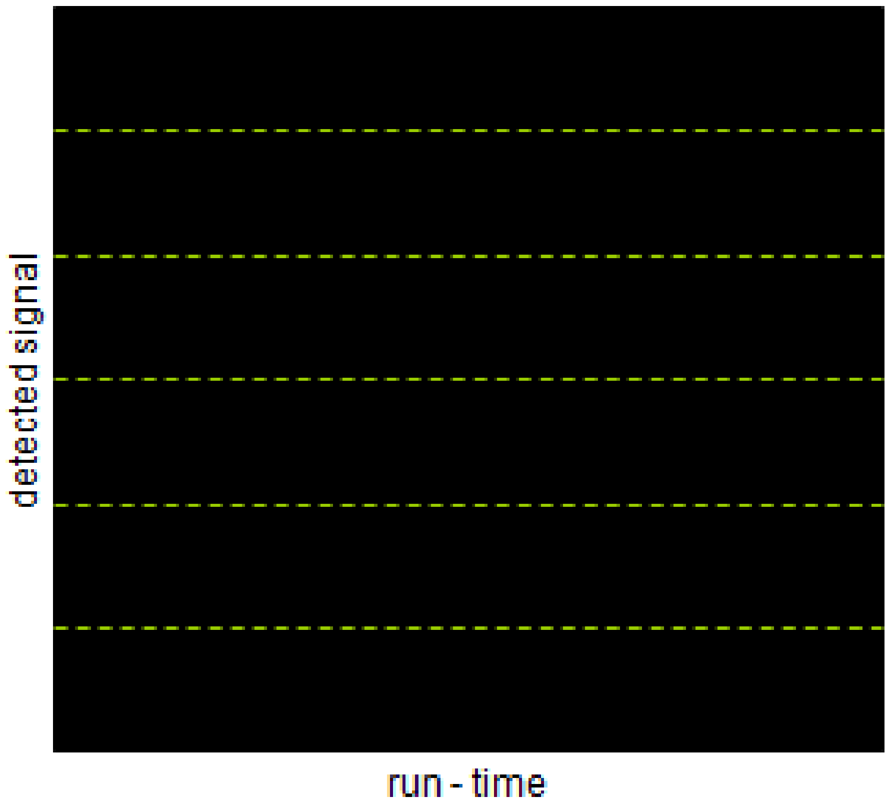
$T(x,y,t)$



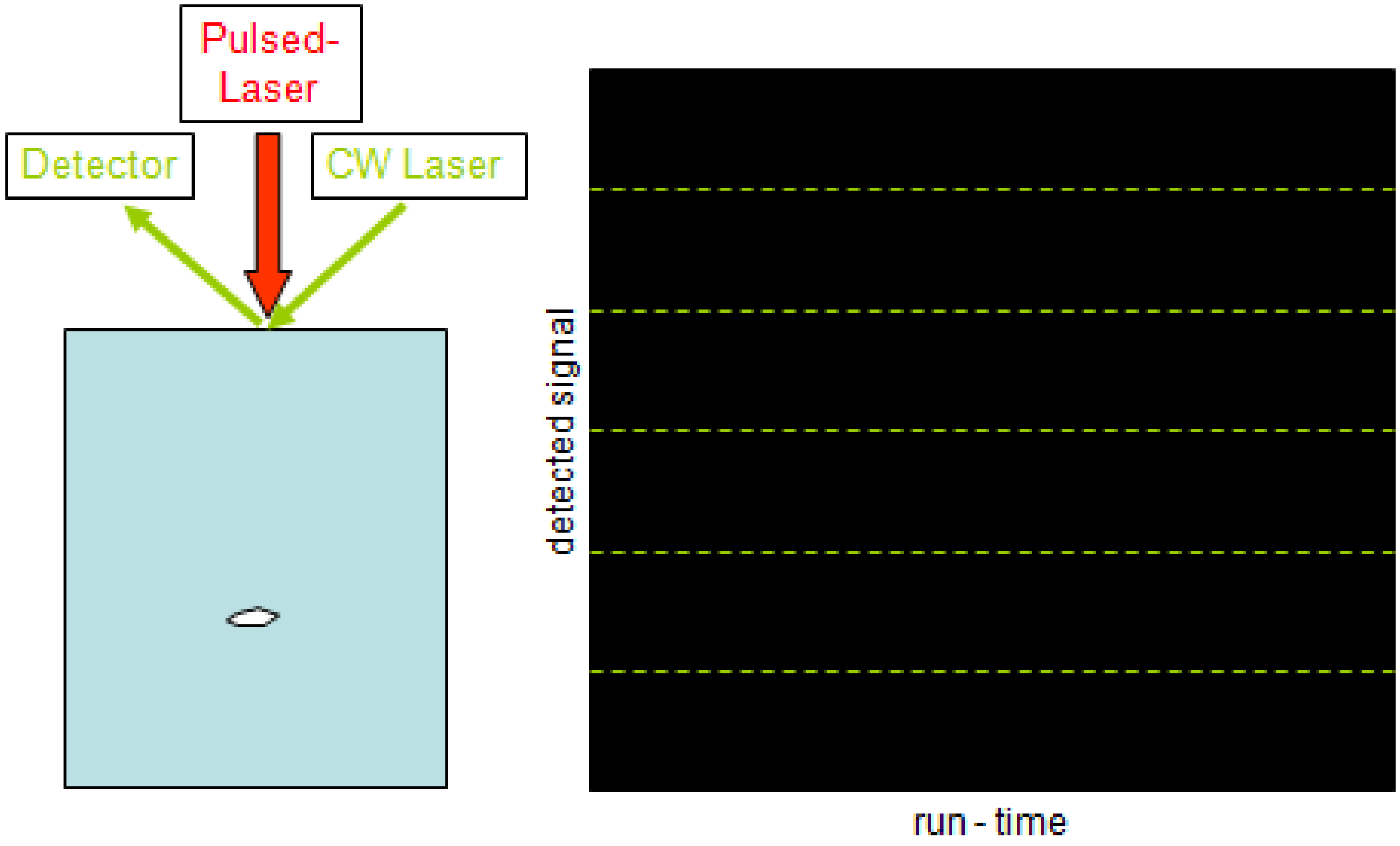
$u_y(x,y,t)$

Laser - Ultrasonics

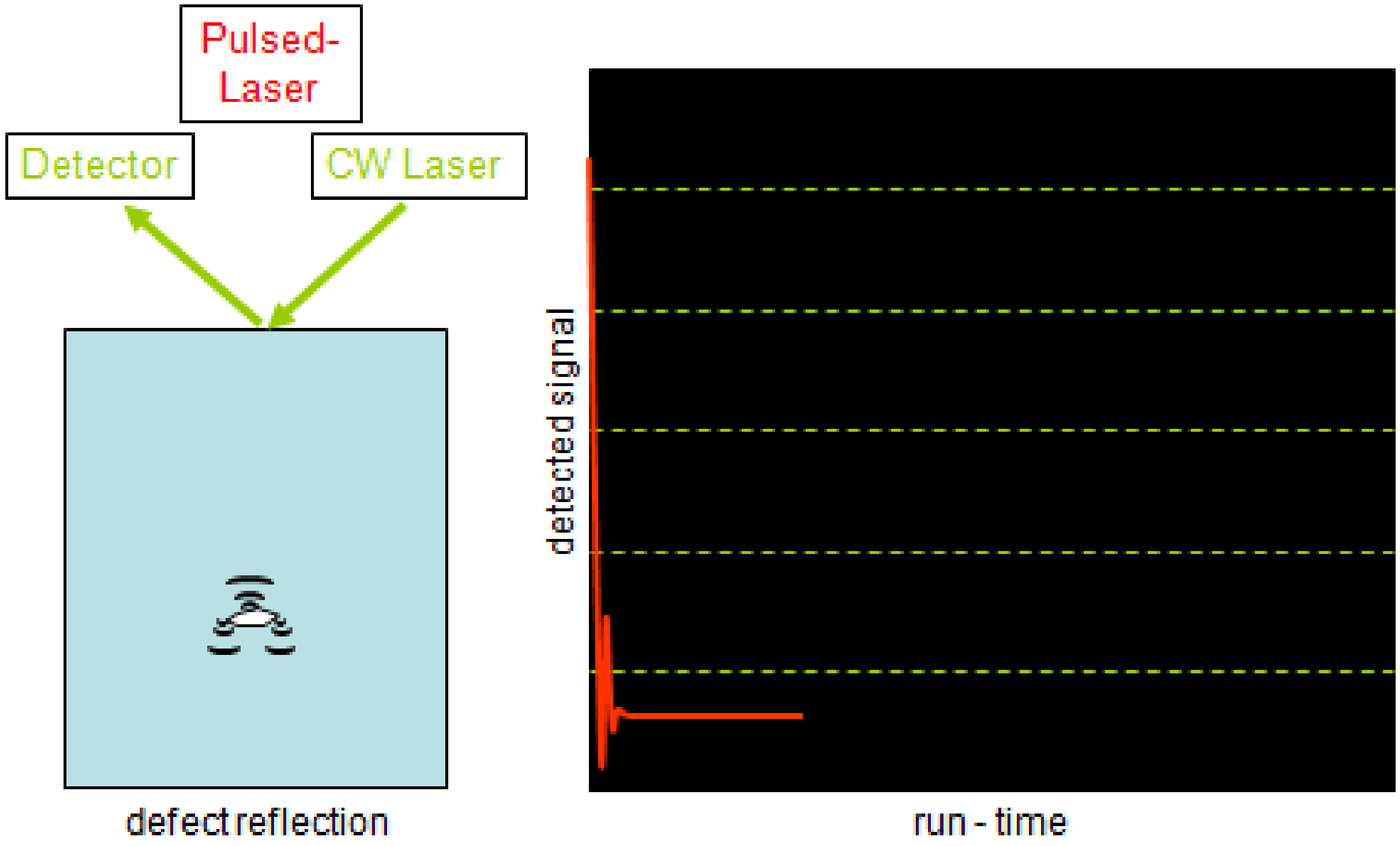




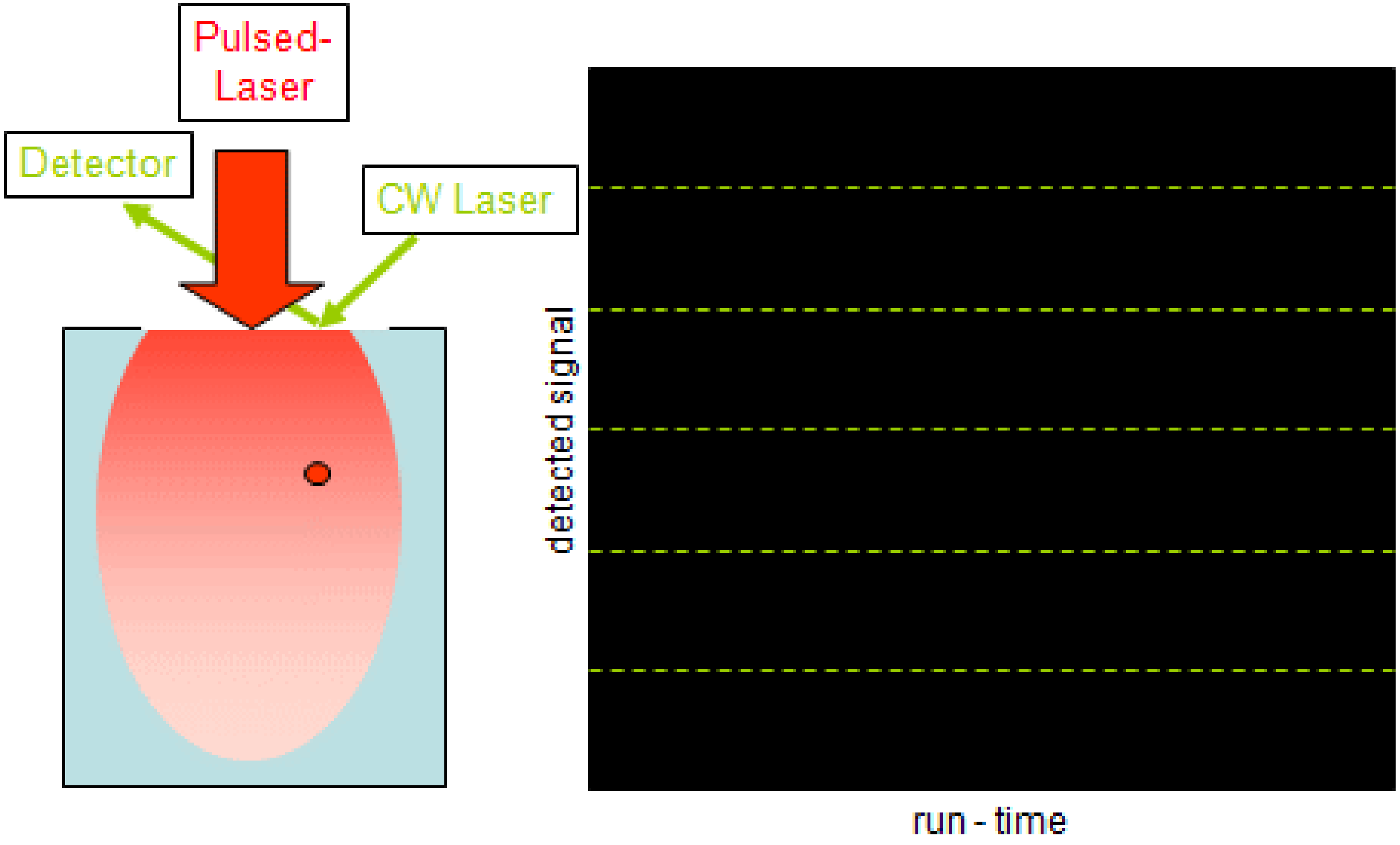
Laser - Ultrasonics



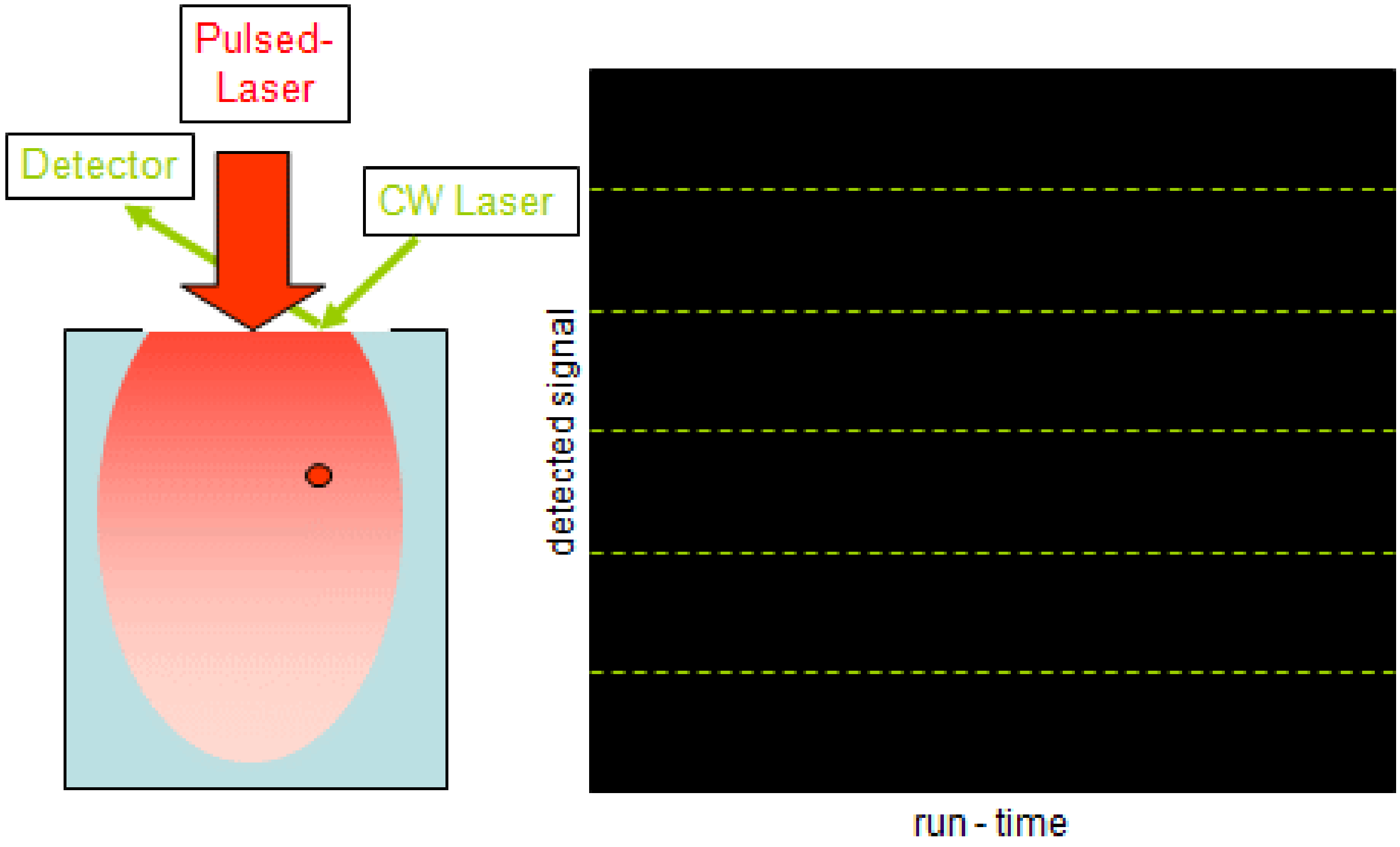
Laser - Ultrasonics



Photoacoustic Imaging

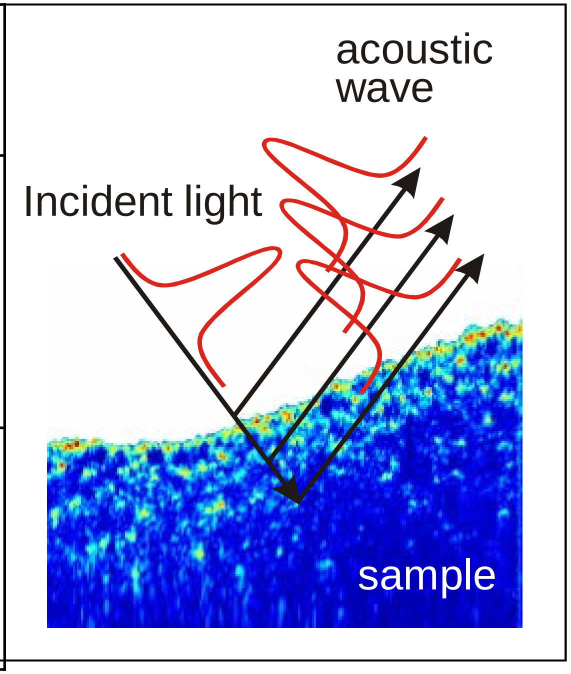


Photoacoustic Imaging

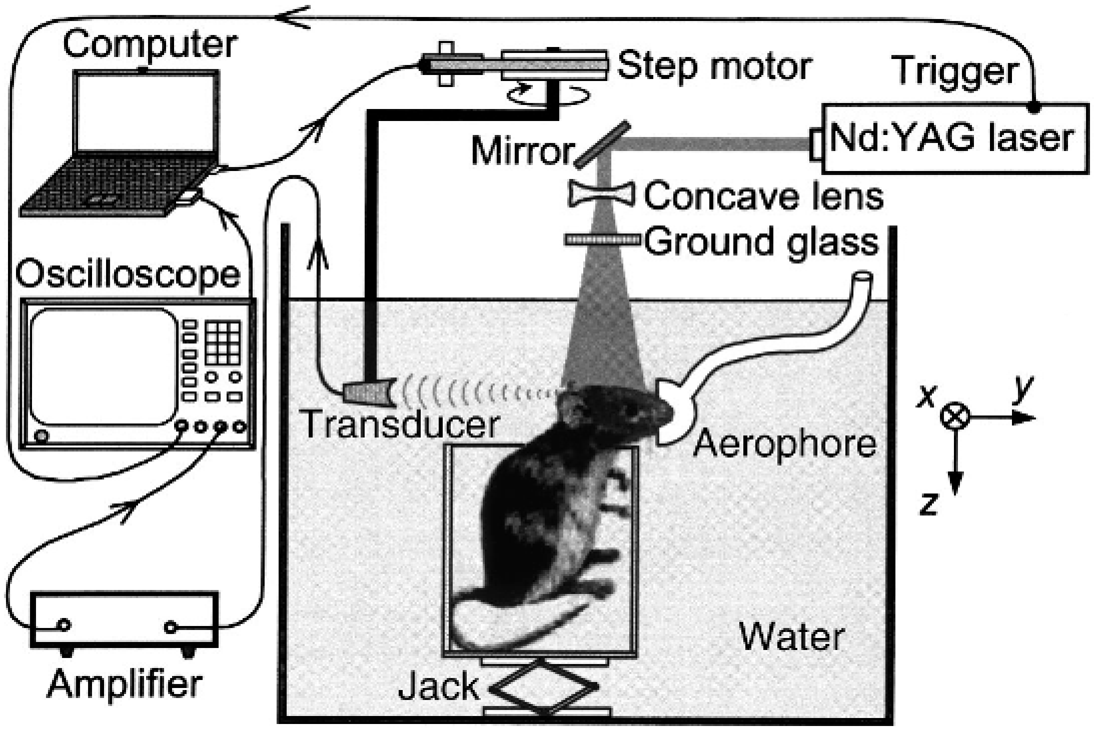


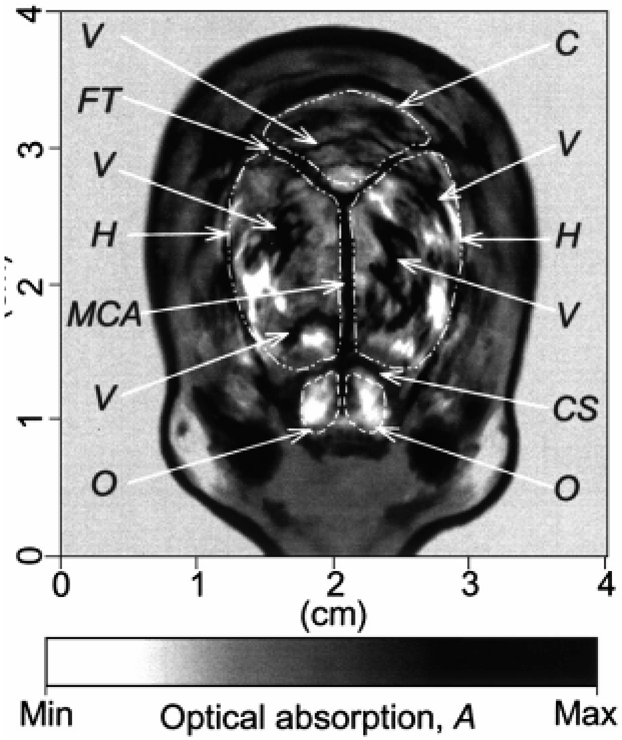
Principle of Laser Ultrasonics and Photoacoustics

	Optical absorption of sample	Acoustic impedance of sample
Laser-Ultrasonics	High absorption, therefore acoustic wave is generated at the sample surface	Contrast is determined by the varying acoustic impedance
Photo-acoustics	Varying absorption coefficient determines contrast	Assumed to be constant (at least in most publications)

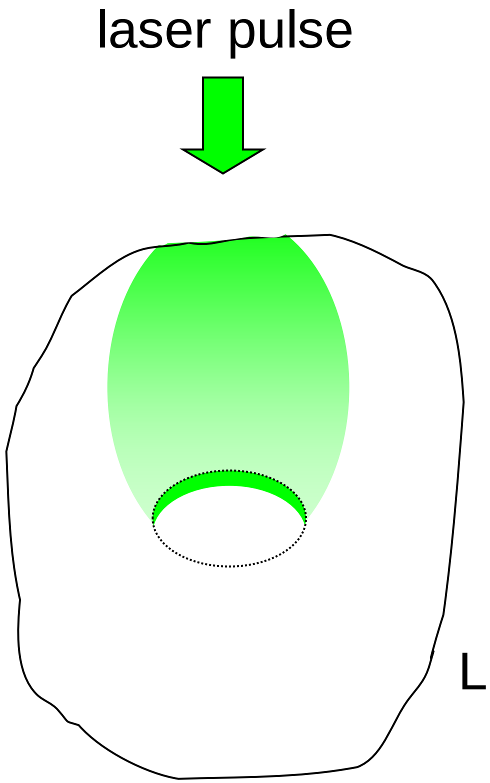


X. Wang, Y. Pang, G. Ku, X. Xie, G. Stoica, and L.-H. Wang, “Non-invasive laser – induced photoacoustic tomography for structural and functional in vivo imaging of the brain,” Nature Biotechnology, vol. 21 , pp. 803-806, 2003





What is imaged in photoacoustics ?



photoacoustic source:

pressure

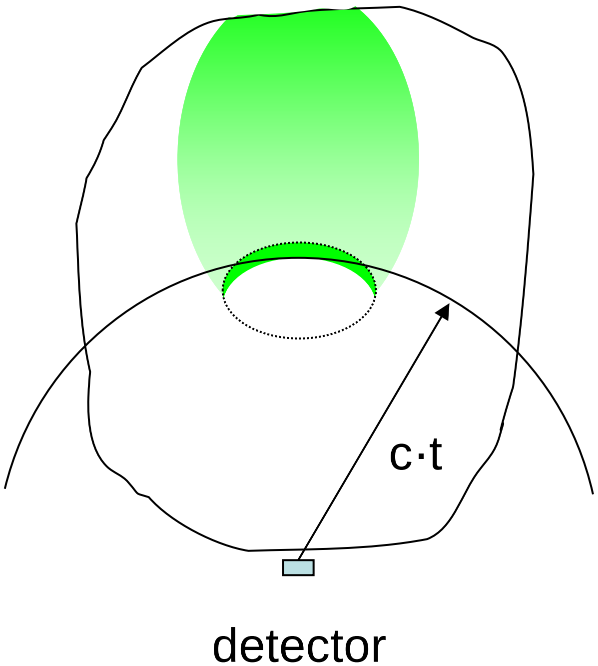
$\propto$

absorbed energy density

=

Light intensity distribution × absorption coeff.

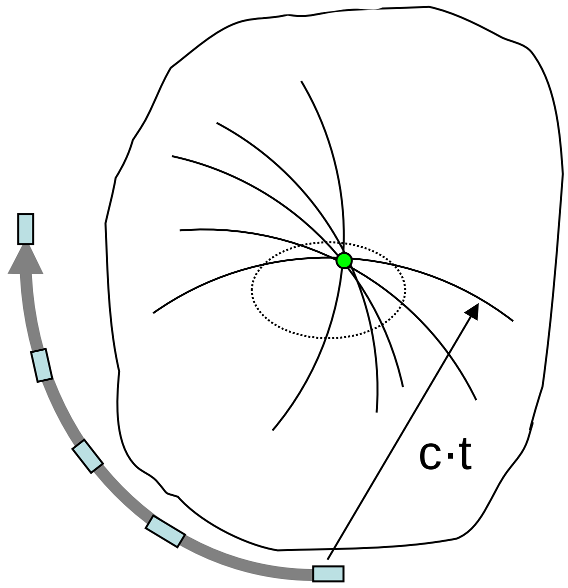
Signal generation



signal at given time t:

integral over surface of sphere with radius c·t

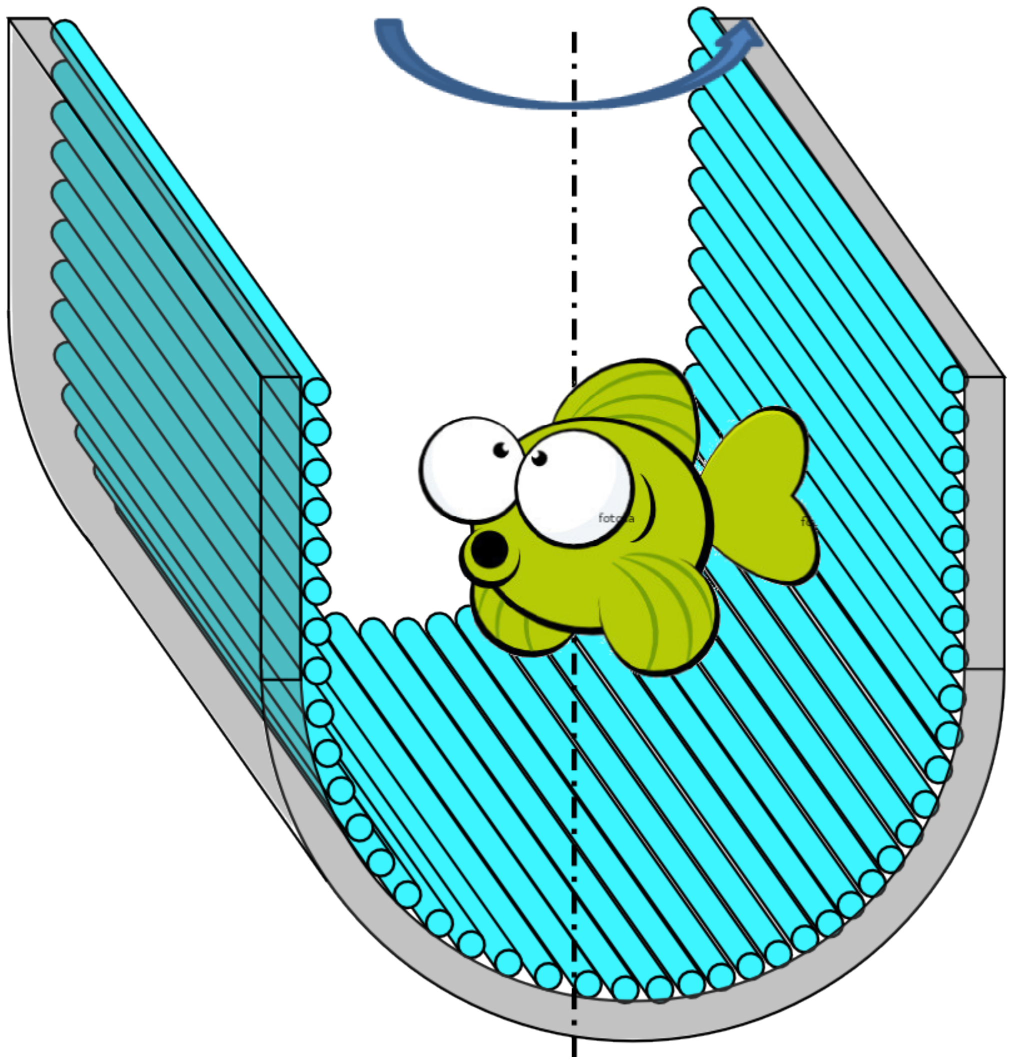
Image reconstruction



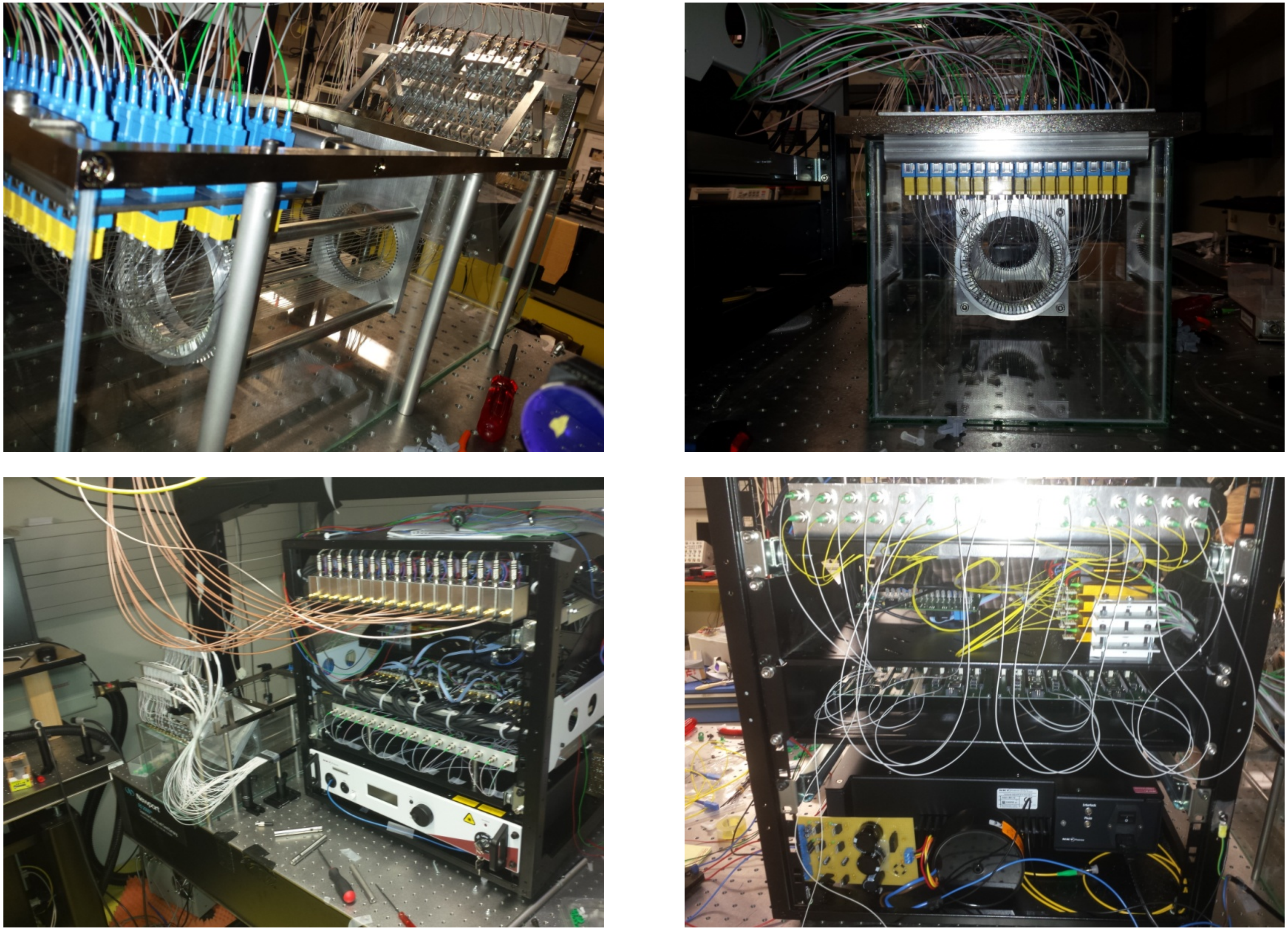
inversion of spherical Radon transform

filtered back projection over spheres

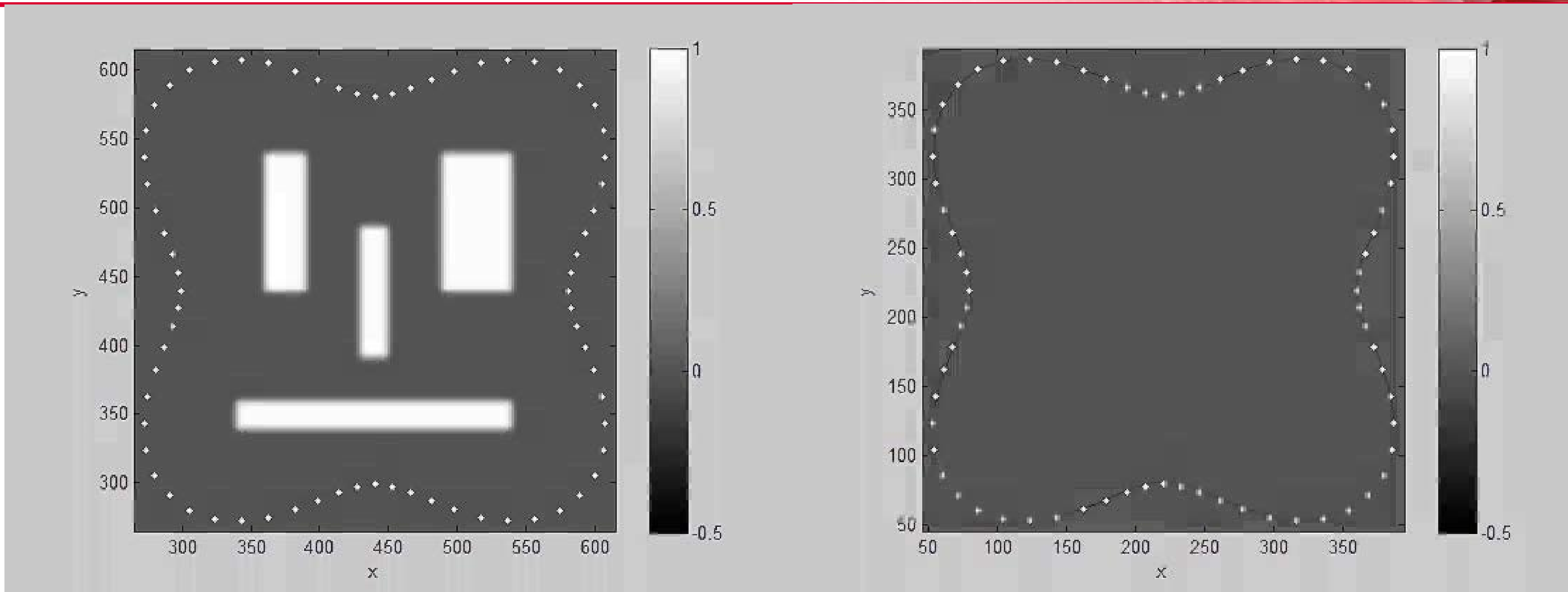
Integrating line detectors

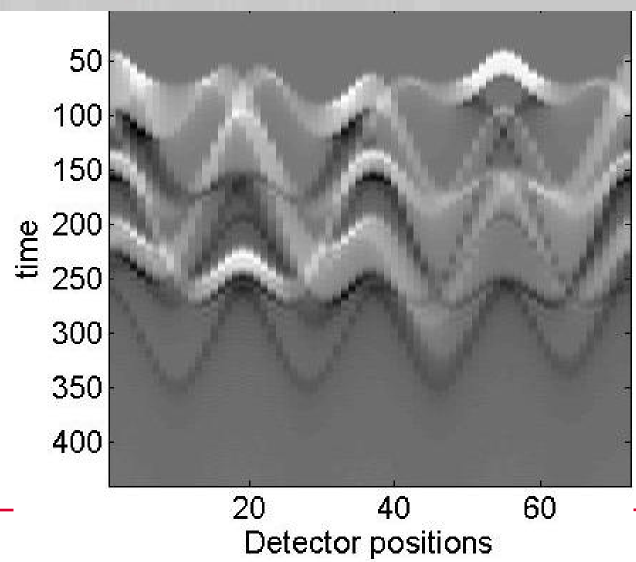


Set-up



Time reversal reconstruction



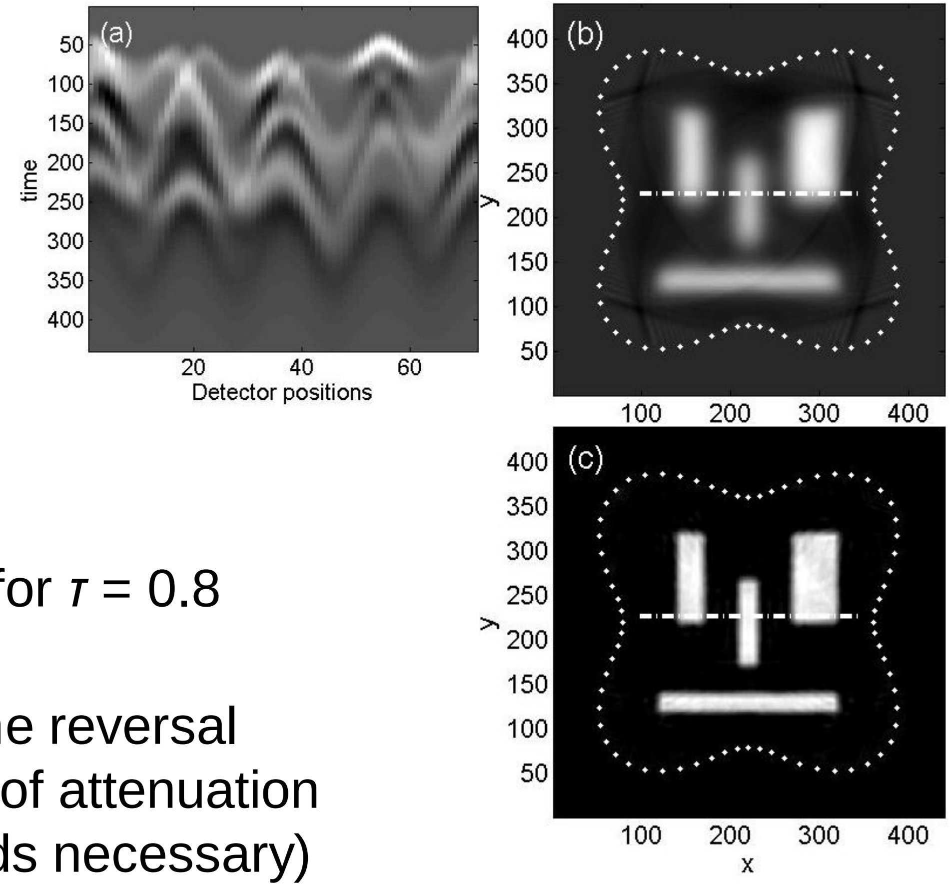


P. Burgholzer, G. J. Matt, M. Haltmeier, and G. Paltauf, “Exact and approximative imaging methods for photoacoustic tomography using an arbitrary detection surface,” Physical Review E 75, 2007

Photoacoustic imaging with time reversal accounting for acoustic attenuation

$$\Delta p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad = 0$$

Stokes equation
Relaxation time τ
Δ…Laplace operator

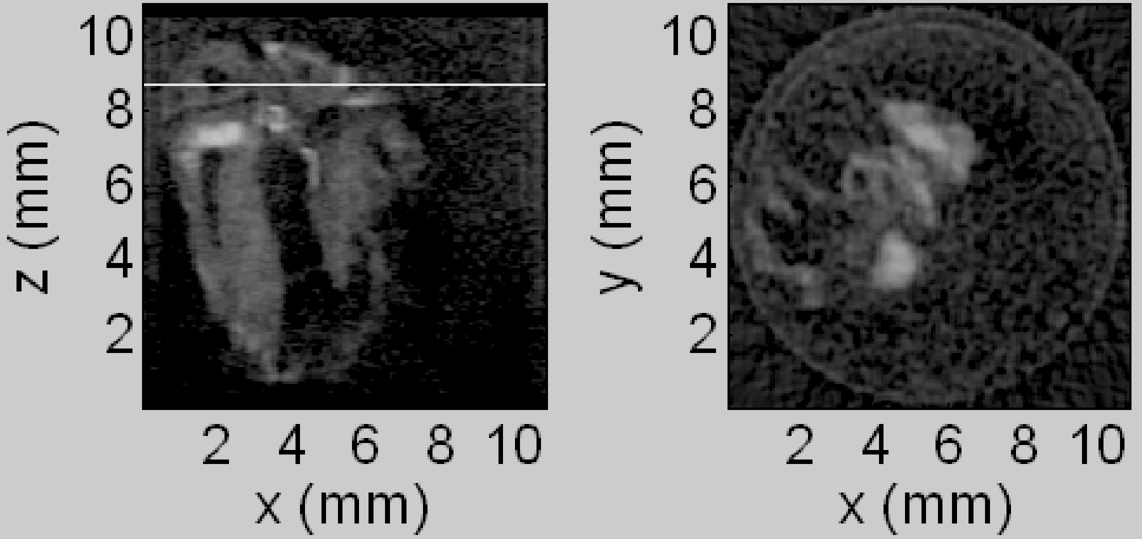


- (a) Pressure simulation results for $\tau = 0.8$ and c=1
- (b), (c) Reconstructions with time reversal without and with compensation of attenuation (ill posed: regularization methods necessary)

P. Burgholzer et al., ”Compensation of acoustic attenuation for high-resolution photoacoustic imaging with line detectors using time reversal” Proc. SPIE 6437-75, Photonics West, BIOS 2007

Photoacoustic tomography of a mouse heart

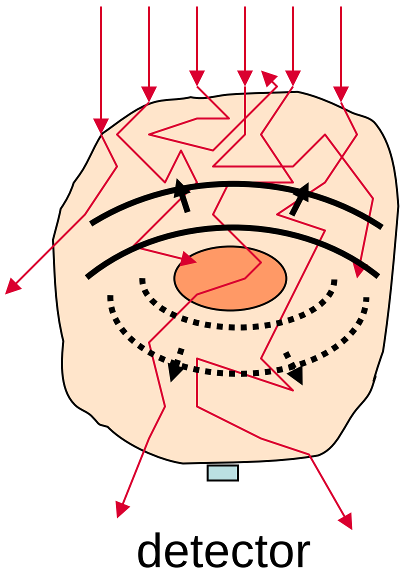
- ***Infrared laser pulse*** generates an ultrasonic wave



- ***Propagated wave*** is measured outside → ***Time reversal reconstruction*** of the inner structure of the heart
- ***Image gets diffuse*** for small structures

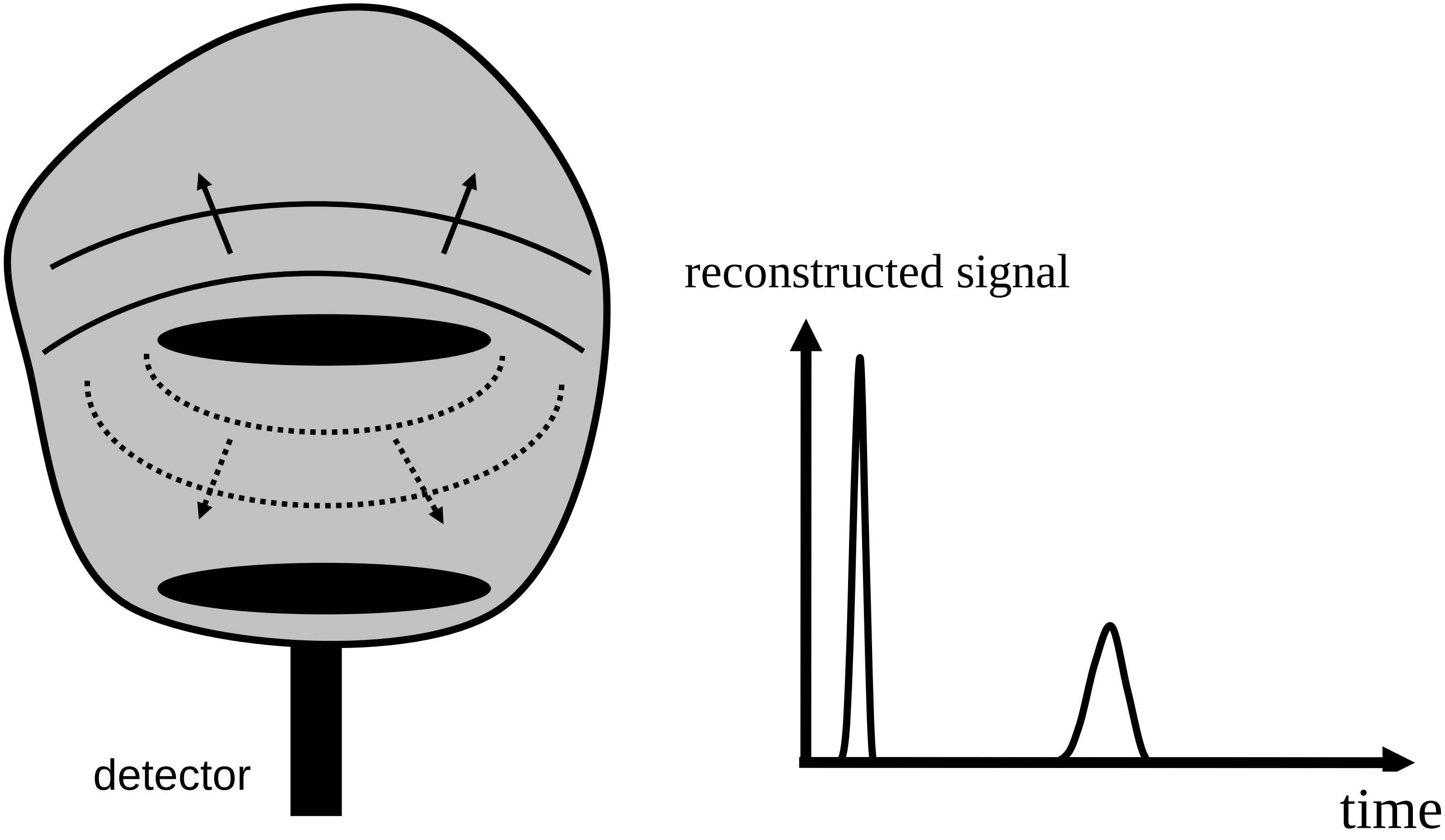
Non-destructive imaging

Spatial resolution is determined by excitation, propagation, and detection of a “wave”, which transports the relevant information to the sample surface: pulse thermography or photoacoustic imaging as an example:

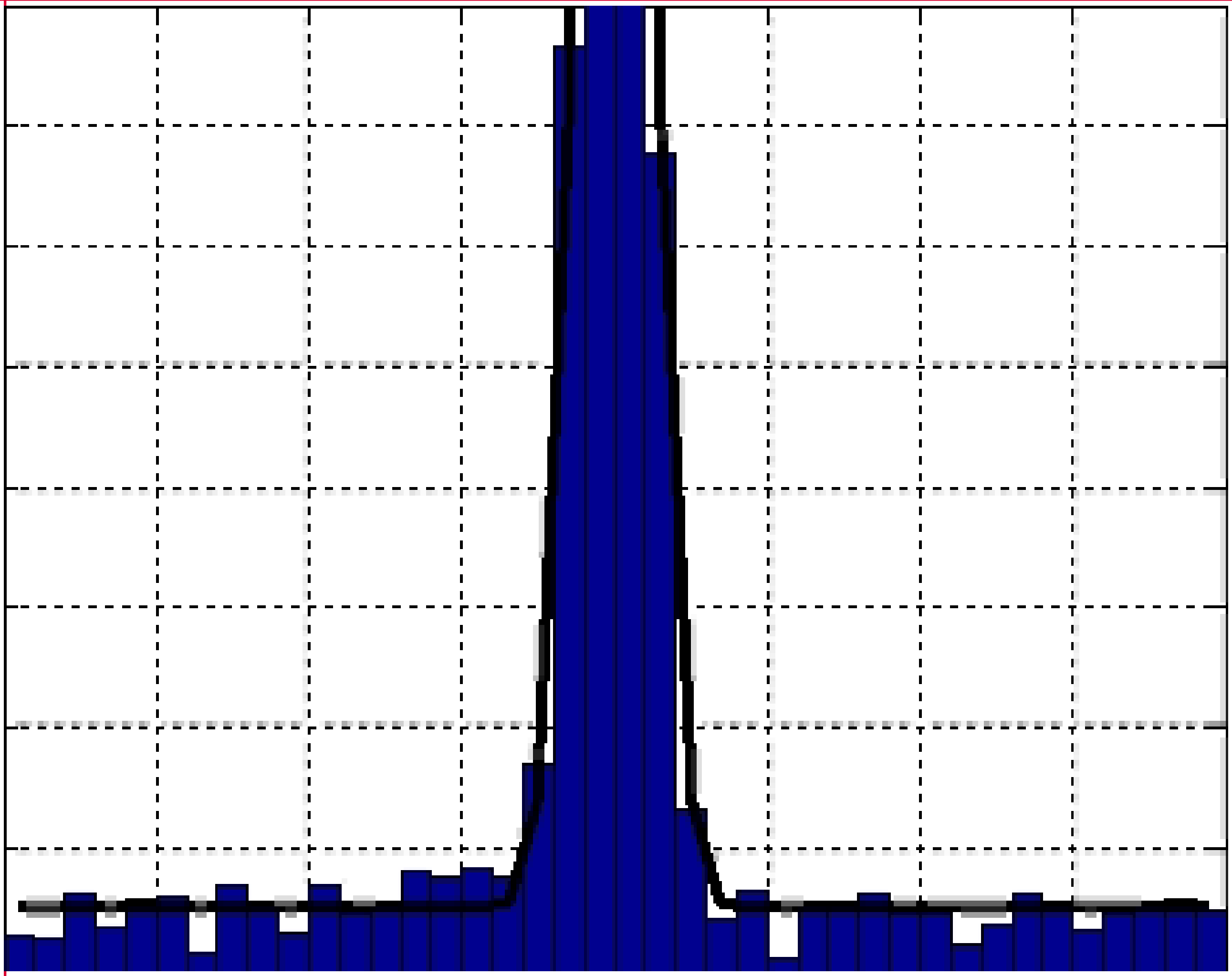


- Generation of a thermal or an acoustic wave: the absorbed light from a flash lamp or a laser heats the structures inside the sample
- Propagation of “waves” to sample surface: diffusion or dissipation (from acoustic attenuation) causes entropy production and a loss of information
- Detection of temperature or acoustic pressure: noise of infrared camera (e.g. shot noise) or bandwidth and size of pressure detector limits spatial resolution

Amplitude Reduction and Broadening



Simple example: heat diffusion in 1D



Diffusion as a stochastic process:

Mean value

Variance

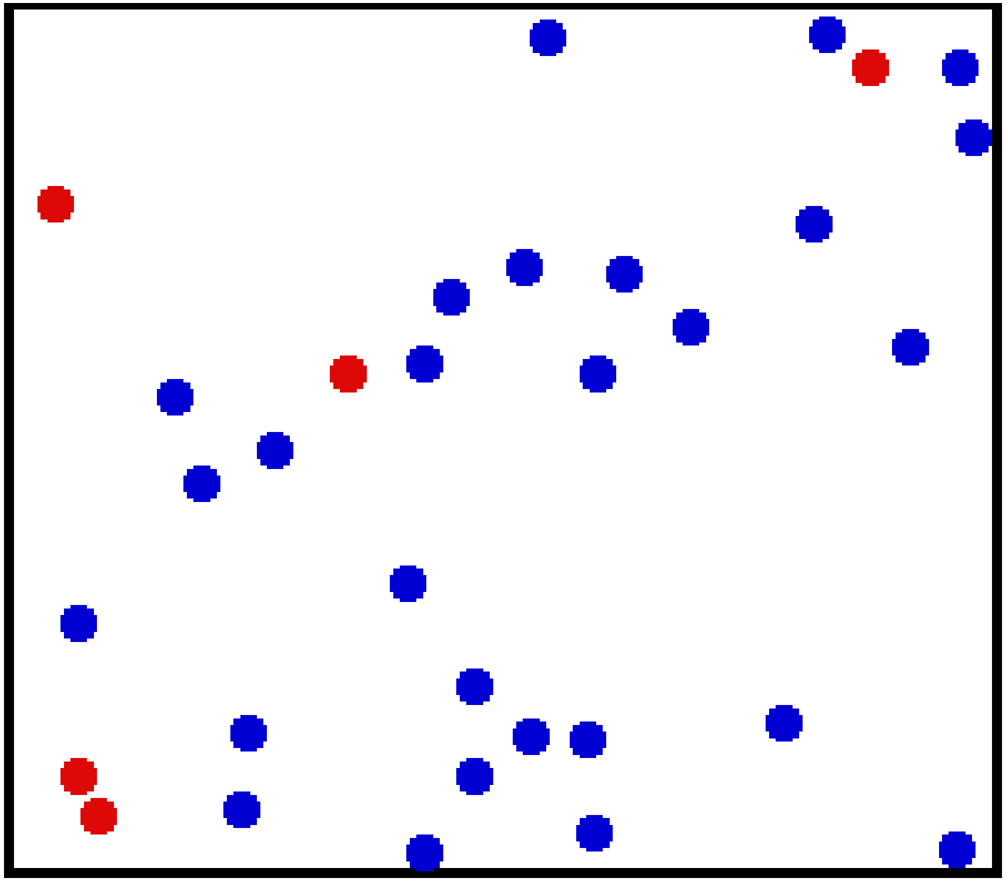
Correlation

Time reversal of a stochastic process:

Initial temperature distribution?

Microscopic picture

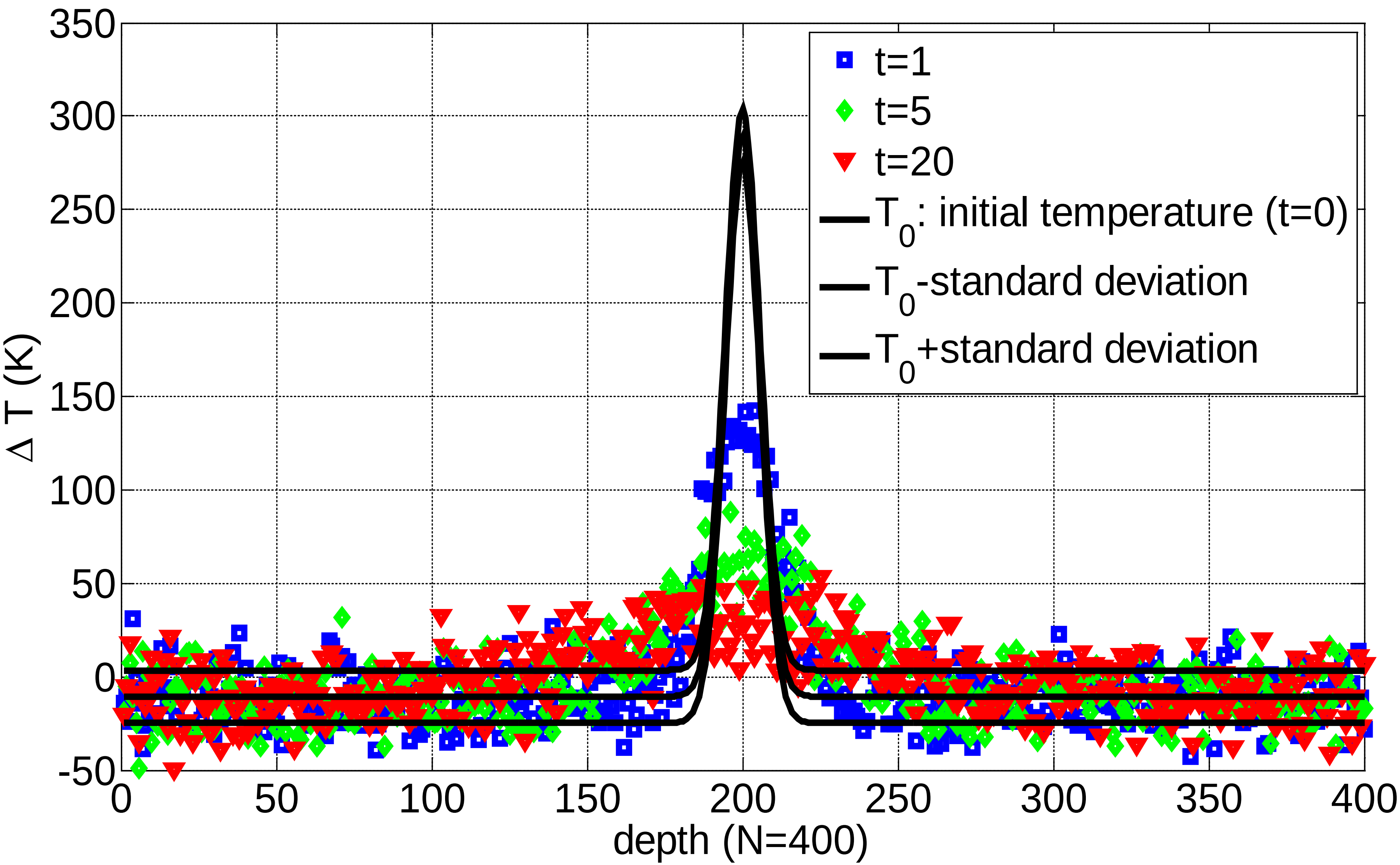
- ***Time reversal*** - fundamental laws of physics are still valid, if the direction of time is changed ($t \rightarrow -t$): e.g. the movement of an ink particle in the water



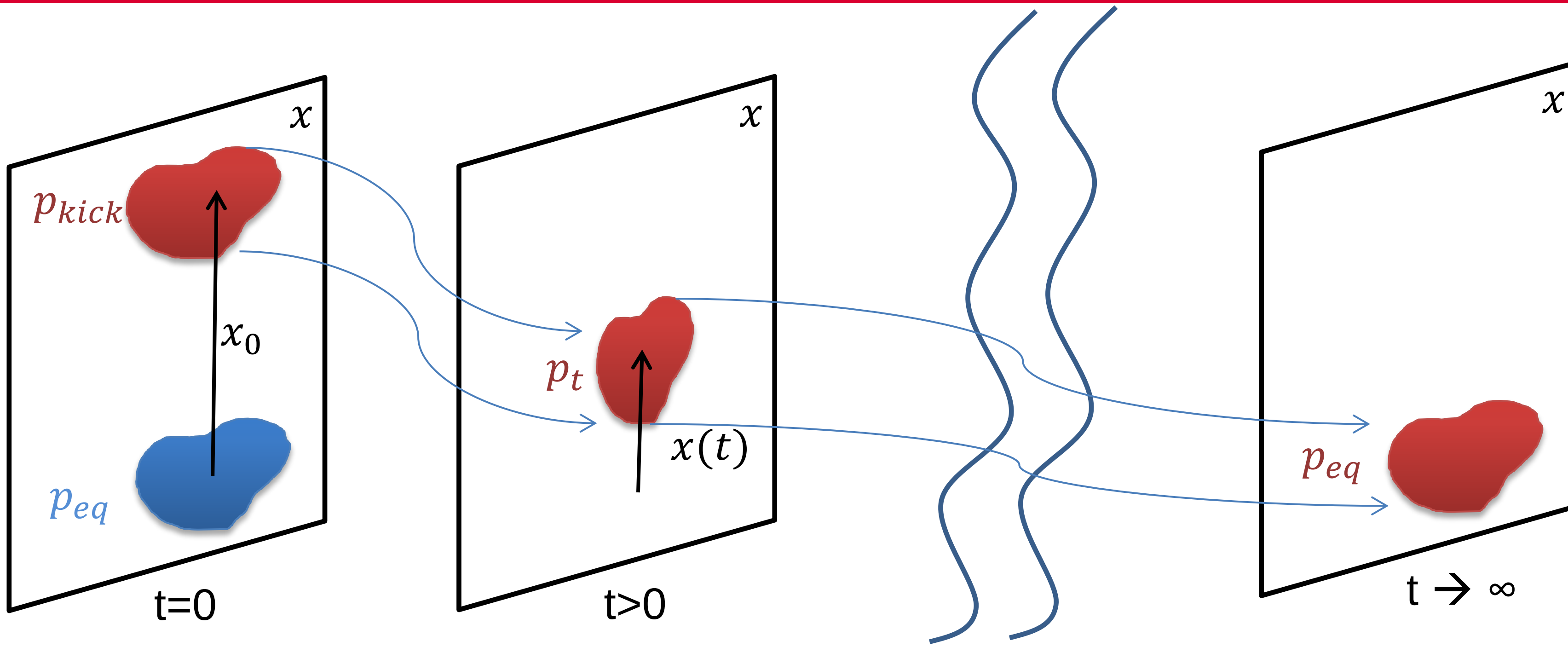
Macroscopic picture of diffusion

- *Drop of ink in water*

Stochastic realization of heat diffusion



"Kicked process" evolving back to equilibrium



Forward process: equilibrium state p_{eq} with mean value at $x = 0$ is kicked at a time t=0 with magnitude x_0 to a state p_{kick} far from equilibrium, followed by a dissipative process back to equilibrium. The arrows indicate the tube of trajectories, which is "thin" for macroscopic systems, as deviations from the mean values $x(t)$ are small. With ongoing time the distance of p_t from is reduced, quantitatively described by a decreasing Kullback-Leibler divergence $D(p_t||p_{eq})$.

Results from Stochastic Thermodynamics

$$I(t) = k_B D(p_t||p_{eq}) \approx \frac{1}{T}\Big(H\big(x(t)\big) - H(x=0)\Big)$$

$$\approx \Delta S\big(x(t)\big)$$

with $D(f||g) := \int \ln(\frac{f(x)}{g(x)}) f(x) dx$

Information about kick magnitude = mean work, which has not been dissipated yet, divided by the temperature = difference in mean entropy.

Chernoff-Stein Lemma:

type II error $\varepsilon = \exp(-nD(f||g))$ if *n* data from g are given, for n large.

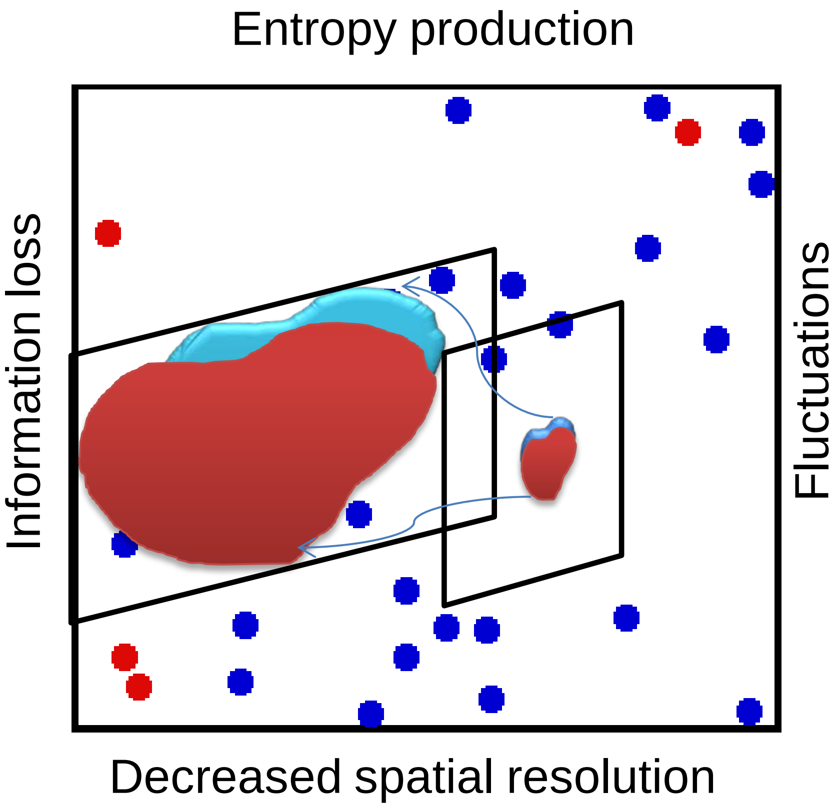
→ p_t cannot be distinguished from p_{eq} if

$$D(p_t||p_{eq}) < \ln(1/\varepsilon)/n$$

Information loss = Entropy production

Physical background of regularization for ill-posed inverse problems:

- Forward Problem: information loss equal to entropy production
- Inverse Problem: lost information cannot be gained any more
- Thermodynamic fluctuations are small for macroscopic samples, but in solving ill-posed problems even small fluctuations are highly amplified and this information loss cannot be compensated by any reconstruction algorithm, and causes a principle resolution limit for reconstructed images.



Fourier space

Mean value equation: linear homogeneous differential equation for the measured signal $g(x,t)$; "wave" can be represented as a superposition of wave trains having a certain wavenumber or frequency in Fourier *k*−space or ω−space, respectively:

$$G(k,t) = \int_{-\infty}^{\infty} g(x,t) exp(ikx) dx$$

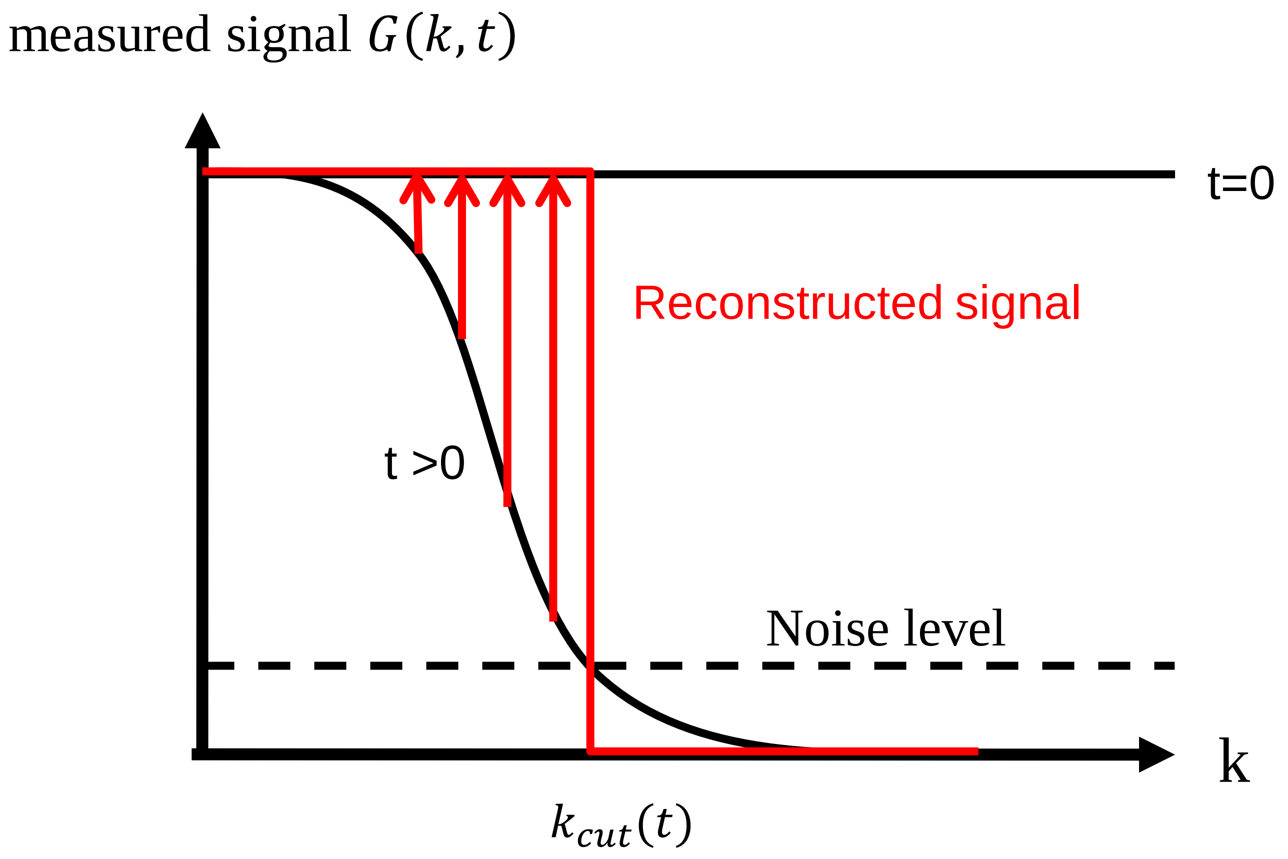
$$g(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(k,t) exp(-ikx) dk$$

where $i = \sqrt{-1}$ and $k = 2\pi/\lambda$.

Thermal diffusion: $G(k,t) = G(k,t)\ exp(-k^2 \alpha t)$

After some time t only wave numbers up to a certain cut-off wave number give additional information. For higher wave numbers D is to small, so they cannot be distinguished from zero.

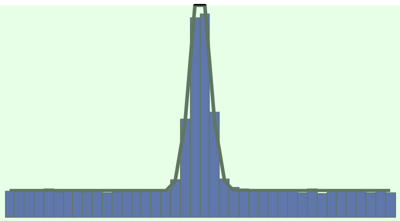
Reconstruction of a Delta-Pulse in k-space



Thermal diffusion in 1D

Real space

$$T_0(x)$$



$$T(x,t)$$

FT

IFT

Time t

Fourier space

“k-space”

$$\hat{T}_k(0) \rightarrow \quad \hat{T}_k(t) = \hat{T}_k(0)\exp(-\gamma_k t) \; with \;\; \gamma_k = \alpha k^2$$

Chernoff-Stein Lemma gives for the highest measureable wavenumber k_{cut}:

$$\exp(\alpha\, k_{cut}{}^2 t) = SNR$$

tSVD and Tikhonov regularization method in k-space (1)

$$\hat{\mathbf{T}}(t) = \hat{\mathbf{T}}(0)\exp(-\boldsymbol{\gamma} t) \; with \; \exp(-\boldsymbol{\gamma} t)) = diag(\exp(-k^2 \alpha\, t))$$

<u>tSVD</u>:

$$\hat{T}_k(0) = \begin{Bmatrix} \hat{T}_k(t)\exp(+\gamma_k t)\,, & for\; k \le k_{cut} \\ 0 & otherwise \end{Bmatrix}$$

<u>Tikhonov</u>:

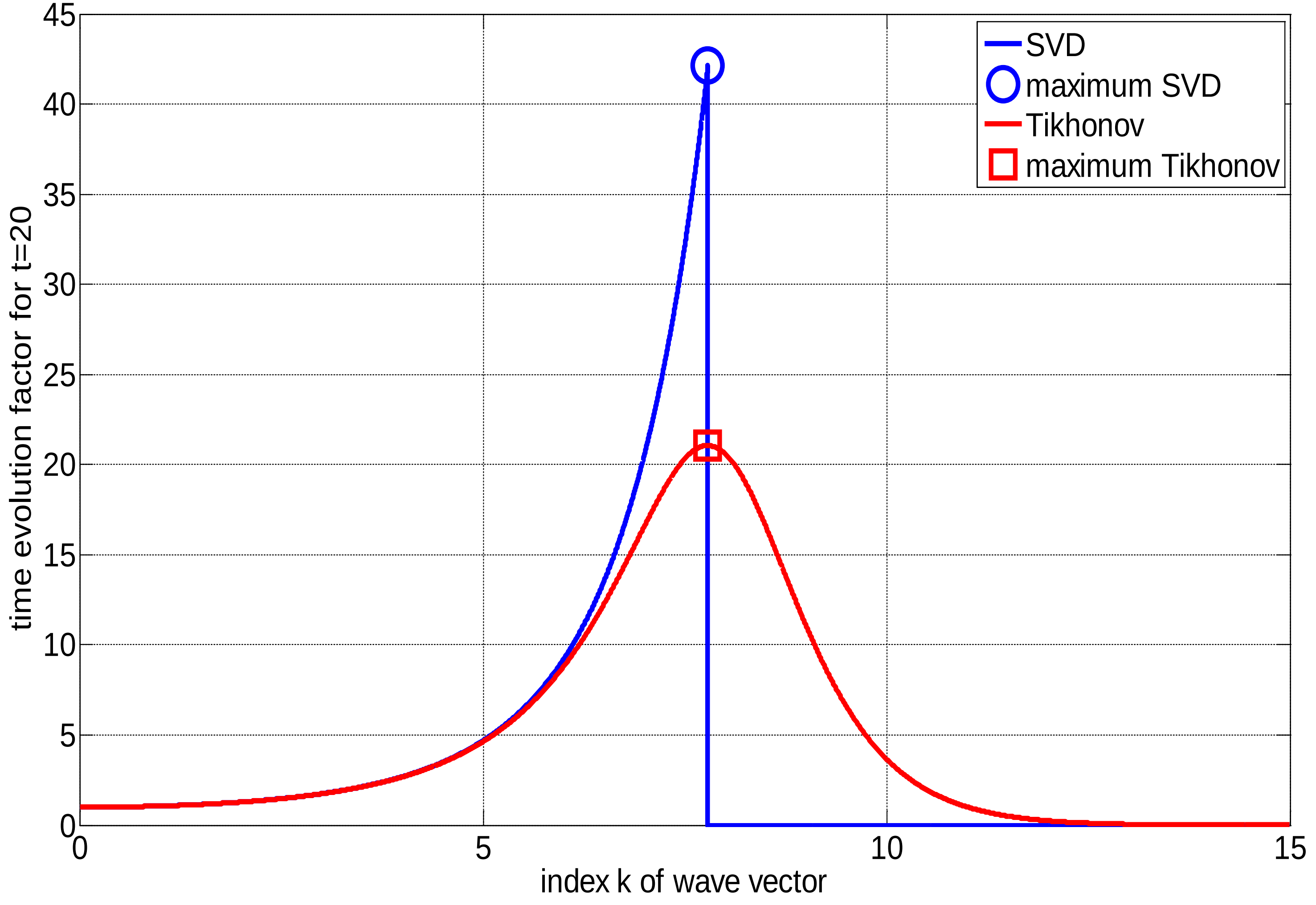
$$\min\{((\hat{\mathbf{T}}(t) - \hat{\mathbf{T}}(0)\exp(-\boldsymbol{\gamma} t))^2 + \lambda \hat{\mathbf{T}}(0)^2\}$$

$$\Rightarrow \hat{T}_k(0) = \frac{\exp(-\gamma_k t)}{\exp(-2\gamma_k t) + \lambda}\hat{T}_k(t)$$

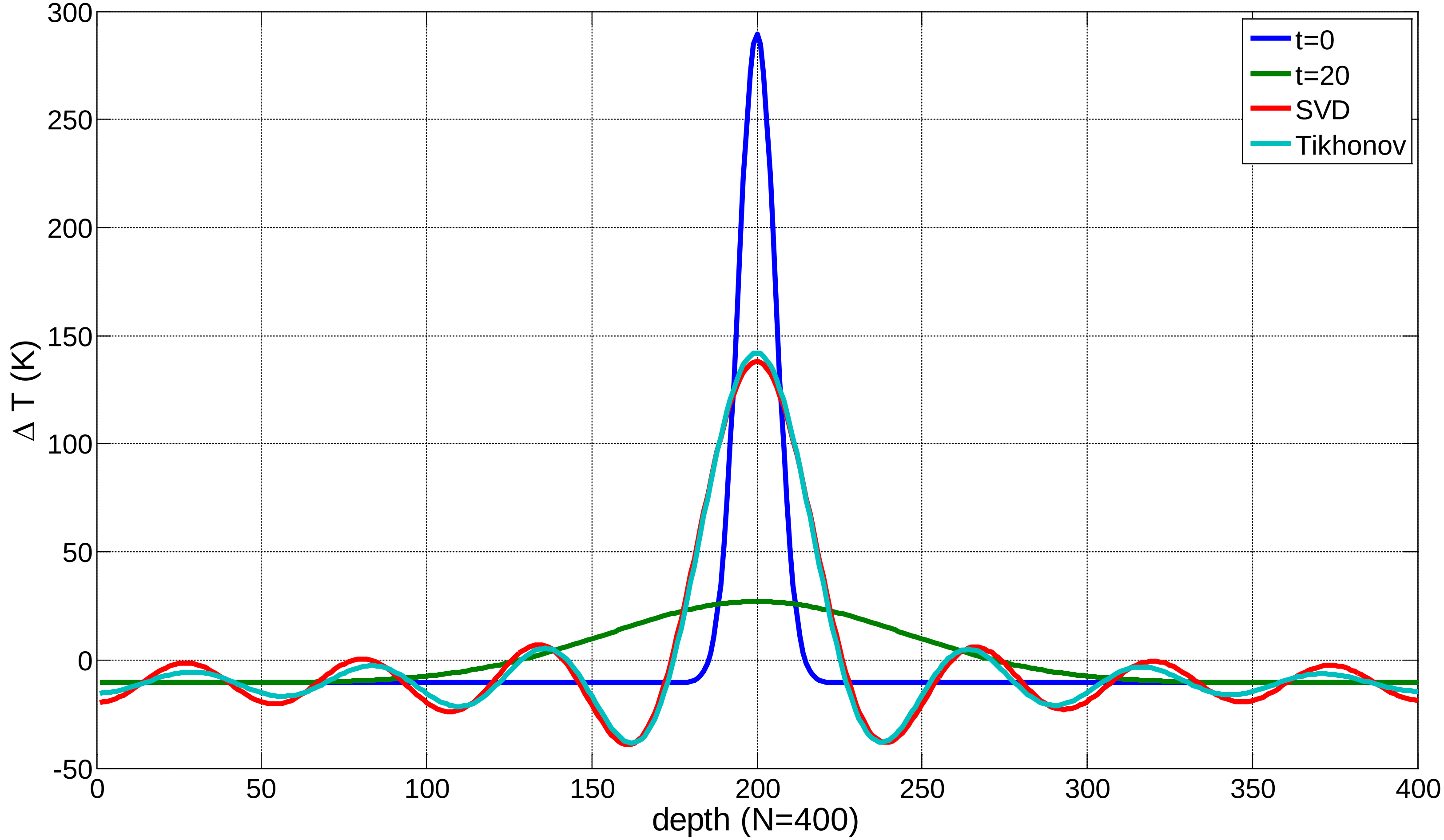
L-curve method: $\lambda = \exp(-2\gamma_{k_{cut}} t) = 1/SNR^2$

$$\Rightarrow \exp(k_{cut}^2 \alpha\, t) = SNR$$

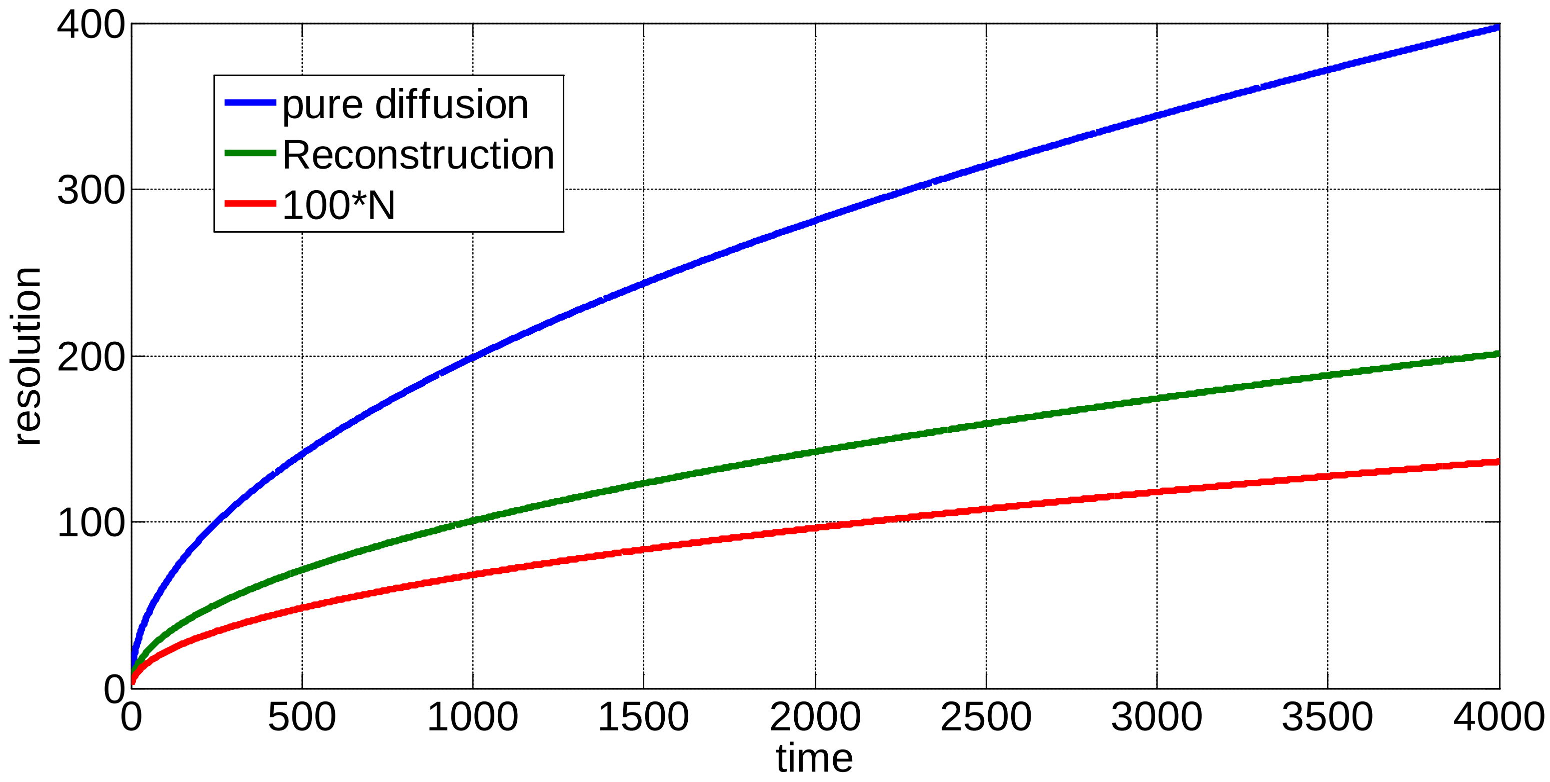
tSVD and Tikhonov regularization method in k-space (2)



Reconstruction of a Delta-Peak

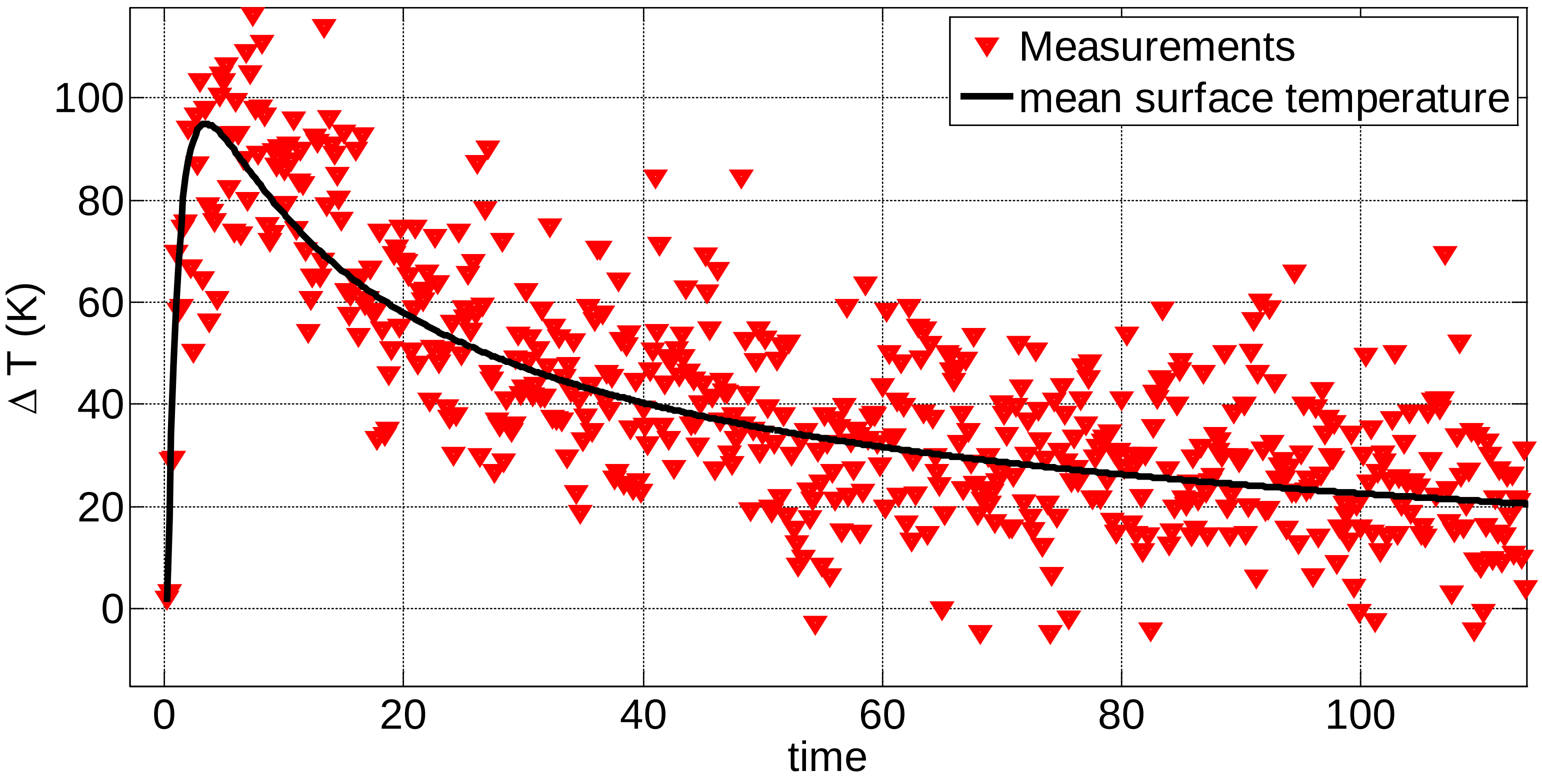


Limits of spatial resolution

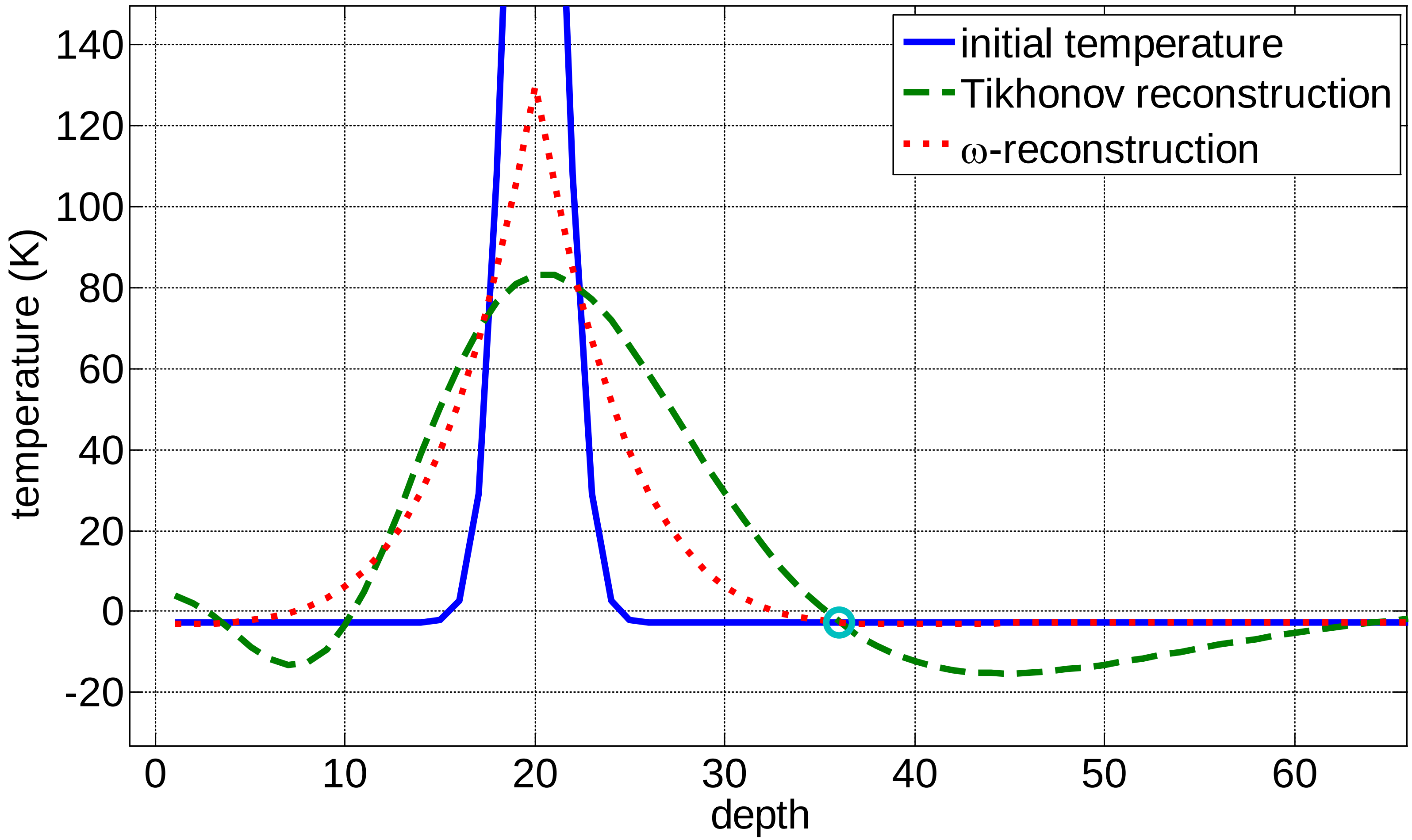


$$\delta(t) = 2\pi\sqrt{\frac{\alpha t}{\ln SNR}}$$

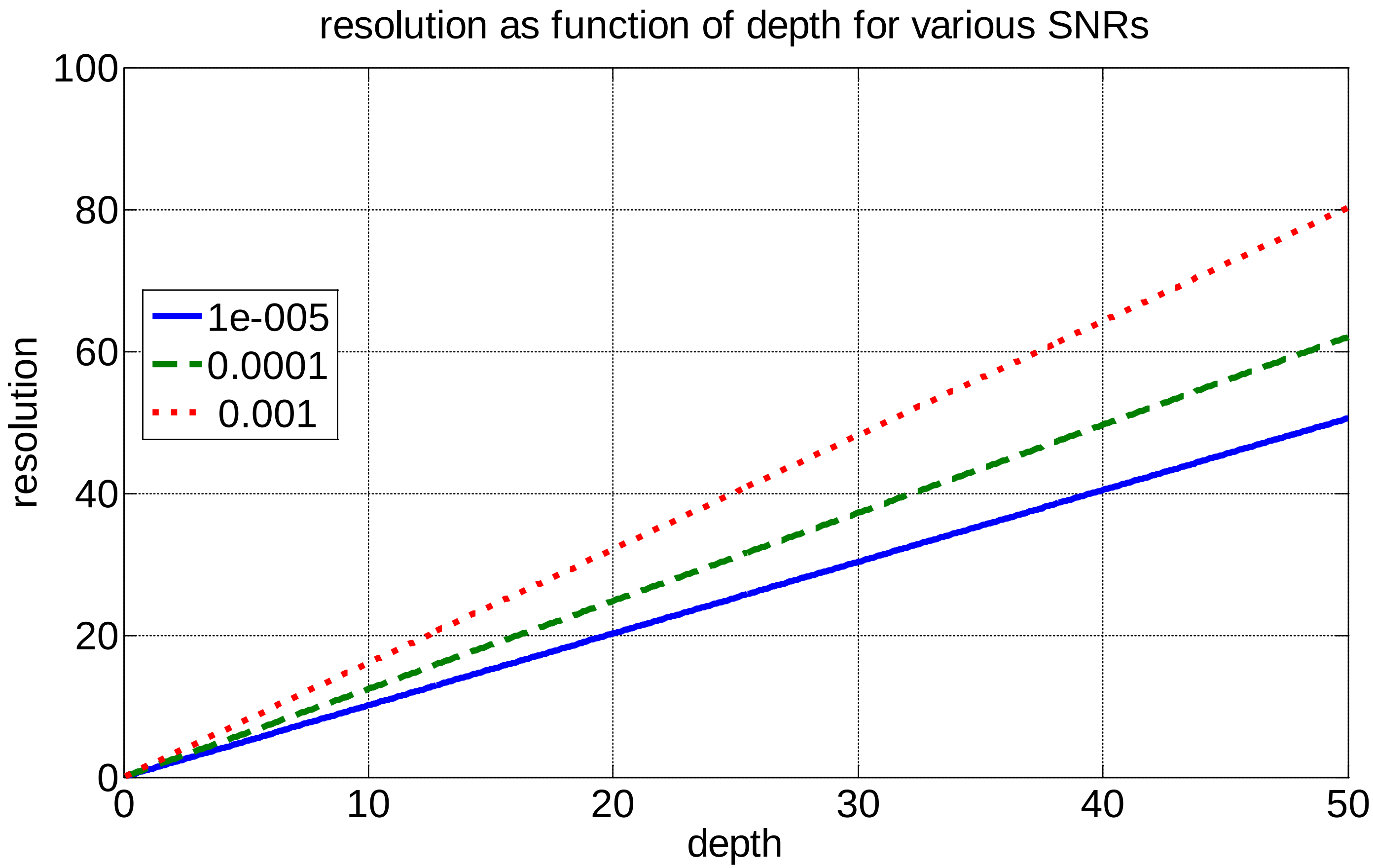
Measured surface temperature



Reconstruction of initial temperature profile



Resolution limit as a function of depth for var. noise levels



$$\delta(a) = \frac{2\pi a}{\ln SNR}$$

Fluctuation-Dissipation theorem

Wave propagation causes

- Entropy production, which can be calculated from measured “mean values” (= dissipated energy / temperature).
- Fluctuation (e.g. thermal noise). Using these fluctuation as a “noise” level the reconstructed image even with the best regularization parameter shows a loss of information (which is equal to the entropy production).

Both are related by a well known relation from Non-equilibrium-thermodynamics:

Fluctuation Dissipation theorem - FDT

Attenuation of 1D “thermal wave”

$$T(x,t) = \mathrm{Re}(T_0 e^{i(\sigma x - \omega t)})$$

inserting in diffusion equation : $\frac{\partial^2}{\partial x^2} T(x,t) = \frac{1}{\alpha} \frac{\partial}{\partial t} T(x,t)$ *gives* :

$$-\sigma^2 = -\frac{1}{\alpha} i\omega$$

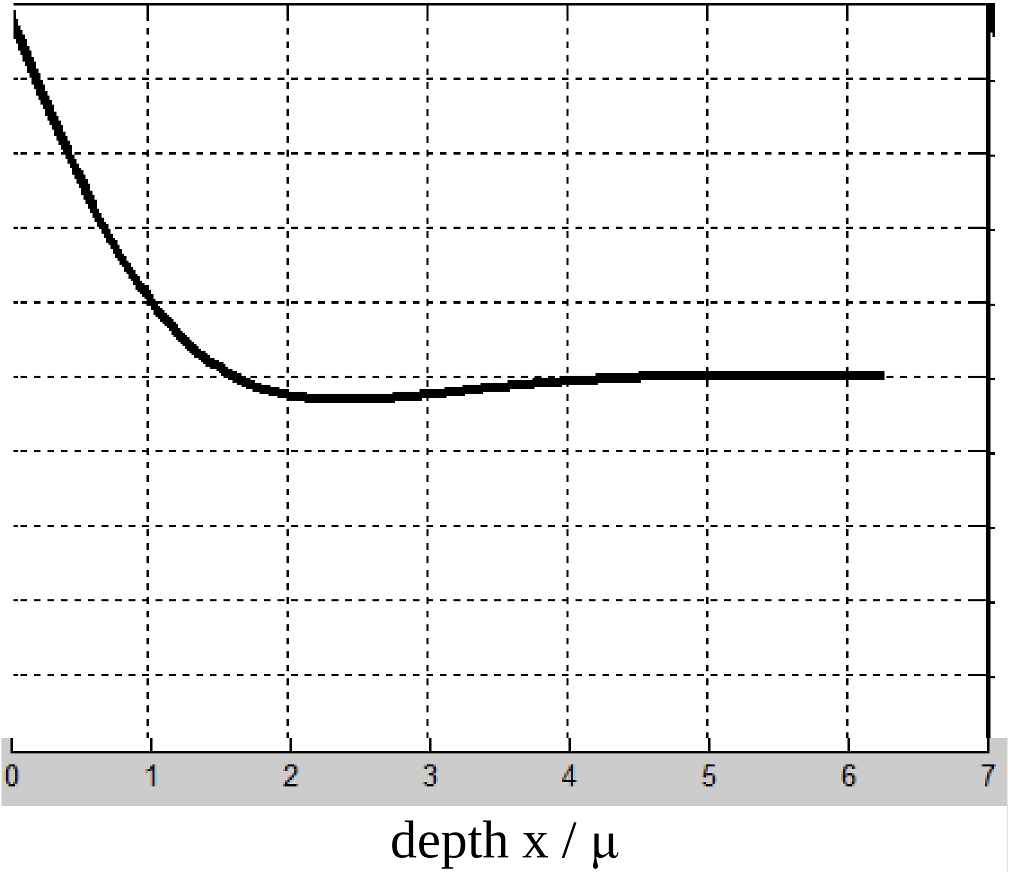
$$\Rightarrow \sigma = \sqrt{\frac{i\omega}{\alpha}} = (1+i)\sqrt{\frac{\omega}{2\alpha}}$$

$$\Rightarrow T(x,t) = T_0 e^{-\frac{x}{\mu}} \cos(\frac{x}{\mu} - \omega t)$$

$$with\ \mu \equiv \sqrt{\frac{2\alpha}{\omega}}$$

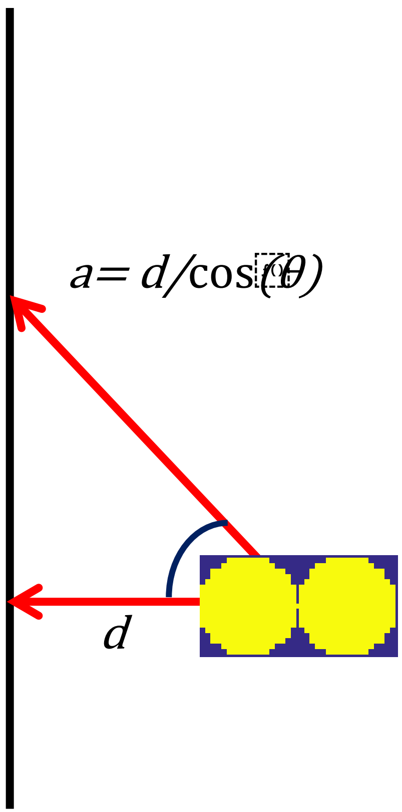
$$wavenumber\ k = \frac{1}{\mu}$$

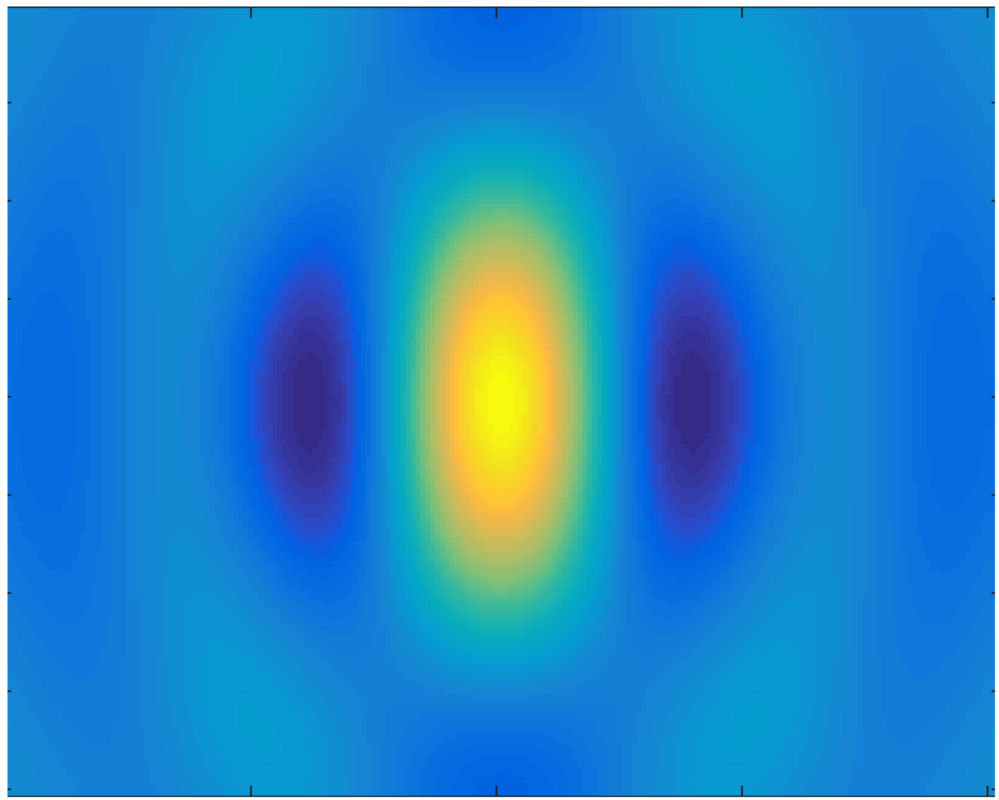
$$\exp(-\frac{a}{\mu}) = \frac{1}{SNR}$$



Thermographic point-spread-function (2D)

$$k_{cut} = \frac{\ln SNR}{a} = \frac{\ln SNR}{d} \cos(\theta)$$





NDTonAIR: PhD position on "Thermographic Reconstruction"

NDTonAIR

H2020 Marie Skłodowska-Curie European Training Network

The Project   Recruitment   Events   Partners

Univeristy of Perugia

The Project

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 722134

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Thank you for your attention

The Persistence of Memory (Salvador Dali)