

Quantum Electrodynamics in the background of evaporating black holes

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Overview

Quantum vacuum in local quantum field theory

Quantum vacuum and its manifestations: Minkowski space

- ▷ Physical vacuum
- ▷ Vacuum fluctuations
- ▷ Electromagnetic properties

Quantum vacuum and its manifestations: Schwarzschild space

- ▷ Physical vacuum
- ▷ Gravitational properties
- ▷ Electromagnetic properties

Concluding remarks

Quantum vacuum in local quantum field theory

QFT starts with a specification of a set of local field operators \mathcal{A} . This set builds the so-called $*$ -algebra. Elements of this algebra being invariant under the $*$ -involution are physical observables.

With each spacetime domain O one associates a local algebra $\mathcal{A}(O)$ built from all operators having non-vanishing support within O . This net of algebras satisfies certain reasonable properties: isotony, locality and covariance.

A state $|\omega\rangle$ on \mathcal{A} is defined as a linear, normalised and positive functional on elements of the algebra \mathcal{A} .

One can assign a Hilbert space \mathcal{H}_ω of the algebra \mathcal{A} to a state $|\omega\rangle$. This is achieved through the Gelfand-Naimark-Segal construction. If the state is physical, then particles $\{|p\rangle\}$ are elements of \mathcal{H}_ω .

More in R. Haag “Local Quantum Physics. Fields, Particles, Algebras” (1996)

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Physical vacuum

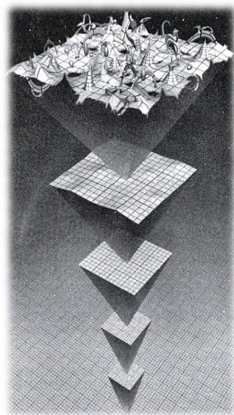
In Minkowski space one can introduce a preferred state - the vacuum state $|\Omega\rangle$, - which is characterised by its invariance under the Poincaré group \mathcal{P}_+^\uparrow and the spectrum condition, i.e. the spectrum of the momentum operator \hat{P}_μ lies in the forward light-cone.

According to Wigner, one associates particle states with the irreducible representations of the Poincaré group. Thus, particle states ($|\text{electron}\rangle, |\text{positron}\rangle$ and so on) are elements of the (Minkowski) Hilbert space \mathcal{H}_Ω .

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Vacuum fluctuations

Quantum vacuum is full of **quantum or vacuum fluctuations**: This immediately follows from the commutation relation imposed on the quantum field operator at different spacetime points. Thus, the field fluctuates and has a non-vanishing value even in a state with no physical particles, i.e. in the vacuum.



Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Vacuum fluctuations

This is fully consistent with **the Reeh-Schlieder property** of the vacuum, of course: there are no local field operators $\mathcal{A}(O)$ with a non-vanishing support within O which annihilate the vacuum $|\Omega\rangle$, i.e. for any $\hat{O} \in \mathcal{A}(O)$ one has $\langle\Omega|\hat{O}|\Omega\rangle \neq 0$, unless $\hat{O} = 0$.

An example of a local observable being one of the elements of $\mathcal{A}(O)$ is a thermometer. Thus, thermometers must measure fluctuations of non-zero temperature even in the vacuum!



Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Vacuum fluctuations

The spectrum of quantum fluctuations is defined via the mean square fluctuation of the field which reads for a massive scalar field

$$\delta(\mathbf{k}) \propto |\mathbf{k}| \times \begin{cases} 1, & |\mathbf{k}| \gg m, \\ (|\mathbf{k}|/m)^{\frac{1}{2}}, & |\mathbf{k}| \ll m. \end{cases}$$

V. Mukhanov, S. Winitzki, "Introduction to quantum effects in gravity" (2007)

The smaller detector, the larger influence of quantum fluctuations.

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Vacuum fluctuations

It follows from **the Tomita-Takesaki theorem**: Whenever one treats a local quantum field operators $\mathcal{A}(O)$ having a non-vanishing support in a given domain O of space, these operators satisfy the so-called KMS condition with respect to a one parametric group of automorphism α_t of the set $\mathcal{A}(O)$, i.e. $\alpha_t: \mathcal{A}(O) \rightarrow \mathcal{A}(O)$.

This group of “time” translations has a geometrical meaning in certain cases, i.e. its generator can be assigned to one of the Killing vectors of spacetime. Examples:

- Bisognano&Wichmann 75': Boost generator in Rindler patch
- Buchholz 78': Dilatation generator in Milne patch

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Electromagnetic properties

The electromagnetic properties of the vacuum are described by electric permittivity ε and magnetic permeability μ appearing in

$$\mathcal{L}_{\text{QED}} = \frac{1}{2} \left(\varepsilon E_i E_i - \frac{1}{\mu} B_i B_i \right) + \text{interaction terms}$$

Integrating the fermion d.o.f. out one obtains an effective action $\mathcal{L}_{\text{QED,eff}}$ for slowly varying field in terms of ε_{ij} and μ_{ij} of the vacuum

$$\begin{aligned}\varepsilon_{ij} &= \delta_{ij} + \frac{e^4}{45\pi m_e^4} \left(2(\mathbf{E}^2 - \mathbf{B}^2)\delta_{ij} + 7B_i B_j \right) + \dots \\ \mu_{ij} &= \delta_{ij} - \frac{e^4}{45\pi m_e^4} \left(2(\mathbf{E}^2 - \mathbf{B}^2)\delta_{ij} - 7E_i E_j \right) + \dots\end{aligned}$$

H. Euler, B. Kockel, Naturwiss. **23**, 246 (1935)

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Electromagnetic properties

In absence of the background electromagnetic field, the non-linear terms do not influence propagation of photons in the leading order approximation of the perturbation theory.

To have a non-trivial effect, one needs either choose a non-vacuum state (assigned to a medium, e.g. plasma) or change somehow the electromagnetic field operators, i.e. \hat{E}_i and \hat{B}_i , structurally.

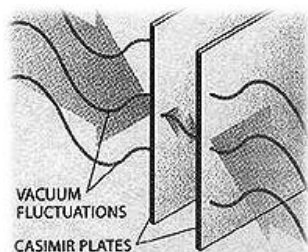
The latter can be achieved, for instance, in-between conducting plates in the Casimir set-up.

Quantum vacuum and its macroscopic manifestations

▷ Minkowski space: Electromagnetic properties

Due to boundary conditions satisfied by the electromagnetic field the field operators $\hat{\mathcal{E}}_i$ and $\hat{\mathcal{B}}_i$ differ from the operators \hat{E}_i and \hat{B}_i defined in Minkowski space!

$$\langle \hat{\mathcal{E}}_i \hat{\mathcal{E}}_j \rangle \neq 0 \quad \& \quad \langle \hat{\mathcal{B}}_i \hat{\mathcal{B}}_j \rangle \neq 0$$



The non-vanishing vacuum expectation value of $\hat{\mathcal{E}}_i \hat{\mathcal{E}}_j$ and $\hat{\mathcal{B}}_i \hat{\mathcal{B}}_j$ entails $\epsilon\mu \neq 1$, i.e. the electromagnetic characteristics of the vacuum modify: Photons propagating perpendicularly to the conducting plates are superluminal. This is **the Scharnhorst effect**.

K. Scharnhorst, Phys. Lett. B236, 354 (1990)

G. Barton, Phys. Lett. B237, 559 (1990)

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Physical vacuum

We await that very, very far away from black holes there should not be any significant imprints of black holes in local experiments performed, e.g., on earth.

We await further that **the physical vacuum** in a black-hole background is that which “goes over” to the ordinary Minkowskian one very, very far from the hole. Therefore, we denote that by the same symbol $|\Omega\rangle$. In other words, the Hilbert space representation \mathcal{H}_Ω does not change when a black hole forms.

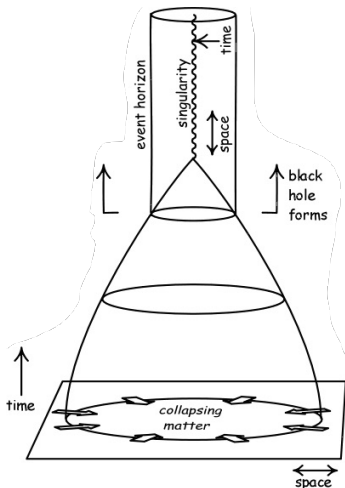
We expect that the notion of a particle should be unchanged independent on the state of motion of an observer, because the Hilbert space representation \mathcal{H}_Ω of the operator algebra is independent on that.

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Gravitational properties

Very far away from a hole, one cannot tell probing the quantum vacuum, whether there is a hole, because the effect is too tiny. It implies gravitational effects due to vacuum fluctuations should be negligible far from the holes.

Indeed, the stress tensor of quantum fields is non-vanishing in the background of black holes, i.e. $\langle \Omega | \hat{T}_\nu^\mu | \Omega \rangle \neq 0$ and vanishes as $1/M^2 R^2$ far from the hole, where M is the black-hole mass.



Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Electromagnetic properties

▷▷ One-loop approximation: Modified propagator

A black-hole horizon entails the modification of quantum field operators. This leads in turn to the modification of the correlation function of the field, e.g. **the propagator of a massless scalar** is

$$G_{\Omega}(k, k') \approx \left(\frac{i}{k^2 + i\epsilon} + \left[\frac{27R_H^2}{16R^2} \right] \frac{2\pi\delta(k^2)}{e^{\beta_H k_0} - 1} \right) \delta(k - k'),$$

where $\beta_H^{-1} = T_H = \frac{M_{\text{Pl}}^2}{8\pi M}$ is the Hawking temperature, $R_H = 2M$, if

$$\begin{array}{ll} R \gg R_H & - \quad \text{weak gravity approximation} \\ |x - x'| \ll R(R/R_H)^{\frac{1}{2}} & - \quad \text{local flatness of spacetime} \end{array}$$

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Electromagnetic properties

▷▷ One-loop approximation: Main observation

Fermion propagator also changes whenever the event horizon is present. This entails

$$\left[\text{Feynman diagram: fermion loop with external wavy lines} \right]_{\text{no BH}} \neq \left[\text{Feynman diagram: fermion loop with external wavy lines} \right]_{\text{BH}}$$

Note that in the case of the Scharnhorst effect, the non-trivial contribution to the photon self-energy appears only at two-loop level, because the fermion propagator does not change in-between the conducting plates - one needs to have internal photon propagator.

Quantum vacuum and its macroscopic manifestations

- ▷ **Schwarzschild space: Electromagnetic properties**
- ▷▷ **One-loop approximation: Small black holes**

Analytic approximation of the propagator is not easy to derive if the fermion is massive. One can “overcome” this technical difficulty by considering small black holes, i.e. $T_H \gg m_e$. This implies

$$M_{\text{Pl}} \ll M \ll 10^{21} M_{\text{pl}} (10^{-17} M_{\odot}) \quad - \quad \text{small black holes}$$

If so, we can neglect m_e with respect to T_H . In other words, fermions are effectively massless (the hard thermal loop approximation).

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Electromagnetic properties

▷▷ One-loop approximation: New temperature parameter

The correction to the fermion propagator induced by a small BH provides with two poles in the photon propagator. This is rather analogous to what happens in the hot physical plasma.

For a basic reference, see: M. Le Bellac, “Thermal Field Theory” (1996)

The hot plasma is however characterised by two parameters, namely the coupling α and temperature T , whereas the black-hole environment is described by one more temperature parameter $\mathcal{T} \ll T_H$:

$$\mathcal{T} \approx \frac{3\sqrt{3}}{16\pi} M_{\text{Pl}} \frac{L_{\text{Pl}}}{R} = 2.34 \times 10^{-7} \left(\frac{1 \text{ km}}{R} \right) \text{ K}$$

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Transverse pole – photon**

Long-wavelength regime:

$$k \ll eT: \omega_t^2 \approx \omega_p^2 + \frac{6}{5}k^2, \quad \text{where} \quad \omega_p^2 \equiv \frac{1}{9}\alpha T^2$$

Short-wavelength regime:

$$k \gg eT: \omega_t^2 \approx m_\gamma^2 + k^2, \quad \text{where} \quad m_\gamma^2 \equiv \frac{1}{6}\alpha T^2$$

ω_p – plasma-like frequency

m_γ – gauge invariant photon mass

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Longitudinal pole – plasmon**

Long-wavelength regime:

$$k \ll eT: \omega_l^2 \approx \omega_p^2 + \frac{3}{5}k^2 \quad - \quad \text{propagating mode}$$

Short-wavelength regime:

$$k \gg eT: \omega_l^2 \approx k^2 \left(1 + 4e^{-k^2/m_\gamma^2}\right) - \text{non-propagating}$$

ω_p – plasma-like frequency

m_γ – gauge invariant photon mass

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Effective photon mass m_γ**

Thus, the effective photon mass due to virtual e^-e^+ pair is

$$m_\gamma^2 \approx \frac{27\alpha}{384\pi} M_{\text{Pl}}^2 \left(\frac{L_{\text{Pl}}}{R} \right)^2$$

According to Coulomb-law tests m_γ cannot be larger than 10^{-14} eV.

E.R. Williams, J.E. Faller, H.A. Hill, Phys. Rev. Lett. **26**, 721 (1971)

Exploiting this result as the upper bound on m_γ , one has

$$m_\gamma \lesssim 10^{-14} \text{ eV} \quad \implies \quad R \gtrsim 252 \text{ km}$$

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: No plasmon excitations**

The Compton wavelength λ_γ of the photon of mass $m_\gamma \lesssim 10^{-14}$ eV:

$$\lambda_\gamma = \frac{h}{m_\gamma c} \gtrsim 1.24 \times 10^5 \text{ km}$$

This is much bigger than $R \gtrsim 252$ km. In the hot physical plasma, plasmons propagate if $\omega_l \sim \omega_p$ or, in other words, $k \ll e\mathcal{T}$. Hence, there are no plasmon-like excitations in the black-hole environment.

This is consistent with the fact that there is no mediator whose collective excitation could be interpreted as a plasmon.

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Electromagnetic properties

▷▷ One-loop approximation: Debye-like screening

The black-hole environment can mimic a certain property of the physical plasma, specifically **the Debye screening** of charges.

The potential of a point-like charge q located at the distance R from the small black hole is given by

$$V(r) = \int \frac{d^3p}{(2\pi)^3} \frac{q e^{-i\mathbf{p}\mathbf{x}}}{\mathbf{p}^2 + \pi_L(p_0 = 0, \mathbf{p})} = \frac{q}{r} e^{-r/r_D},$$

where

$$r_D = \frac{1}{\sqrt{2}m_\gamma} \quad - \quad \text{Debye radius}$$

Quantum vacuum and its macroscopic manifestations

▷ Schwarzschild space: Electromagnetic properties

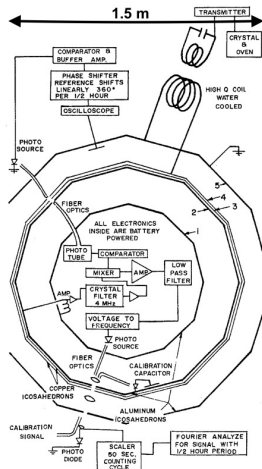
▷▷ One-loop approximation: Debye-like screening

Above we have found that for a photon of mass $m_\gamma \lesssim 10^{-14}$ eV generated by the small black hole, this hole must be at a distance $R \gtrsim 252$ km.

For the same upper bound on the photon mass m_γ , one has

$$r_D \gtrsim 8.77 \times 10^4 \text{ km} \gg R$$

Hence, the point-like **charge q can be only partially screened** due to the vacuum polarization effect.



Concluding remarks

- ▶ The electromagnetic properties of the quantum vacuum changes in the presence of black holes. This is a consequence of the modification of the quantum field operators, rather than the vacuum itself.
- ▶ Photon acquires an effective mass m_γ in the background of black holes which is vanishing far from the small black holes. For large black holes, i.e. $M \gg 10^{16}$ g, this effect is exponentially suppressed as e^{-m_e/T_H} .
- ▶ The collective excitations known from hot plasma physics are impossible in the black-hole environment. This is due to the absence of mediators in the quantum vacuum (no-particle state). In plasma that is provided by the plasma particles.

Concluding remarks

- ▶ However, a point-like charge can be partially screened in the black-hole environment. In plasma that is due to collective response of plasma particles to the static electric field of the probe charge. Near small black holes the Debye-like screening is a vacuum polarization effect.
- ▶ Electrons get a one-loop correction to their mass which is of no significance at the distances $R \gg 4.4 \times 10^{-15}$ m, whereas the classical estimate of the electron size is about 2.8×10^{-15} m.
- ▶ The one-loop correction to the fine structure constant α near the small black holes is also negligible: $\delta\alpha/\alpha \ll 10^{-23}$ at a distance 1 m from the hole.