

# Applicability of the Ni's solution of Einstein field equations to the real objects<sup>1</sup>

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**ABSTRACT.** In 2011, Chinese researcher Jun Ni published the result of his solution of the Einstein field equations for the spherically symmetric distribution of matter. These equations were the same as Oppenheimer and Volkoff (O&V) used in their famous classical work on neutron cores. However, in contrast to the O&V result, Ni obtained the solution, which enables a construction of relativistic compact object (RCO) of whatever a large mass and its outer surface is always situated above the event horizon. The Ni's solution appears to be the super-class of the O&V's solution. The proofs of the maximum mass of stable RCO are valid only for the O&V sub-class. In our contribution, we discuss the main reasons of the differences between the Ni and O&V solutions and suggest some observations of objects and phenomena, which could support or suppress the idea of applicability of the Ni's solution to the real RCOs. Especially, we give a model for an object resembling a quasar with an extended, galactic-scale "corona".

## Ni solution of field equations

In 2011, Ni published the result of his solution of the Einstein field equations (EFEs) for the spherical symmetry in purpose to construct the stable compact objects - the same equations as Oppenheimer and Volkoff (O&V) used in their famous classical work on neutron cores published in 1939.

It appears that the Ni's solution and resulting models of stable compact objects are the super-class of those obtained by O&V. The extent of the set of the Ni's solutions/models and O&V sub-class can be seen in the scheme in Fig. 1.

The O&V sub-class represents only a tiny fraction of all solutions. The current astrophysics of neutron stars and other relativistic compact objects (RCOs) is built on only this tiny fraction of the possibilities offered by the general relativity (GR).

## On the upper mass limit

There was published the proof of maximum mass of neutron star (Rhoades and Ruffini, 1972; another papers with an improvement of specific value of the maximum mass).

If one analyses the proof, then he or she finds that it is valid only for the O&V sub-class of solutions.

Using the Ni's strategy to solve the EFEs, everybody can find many contra-examples of the stable RCOs of whatever large mass.

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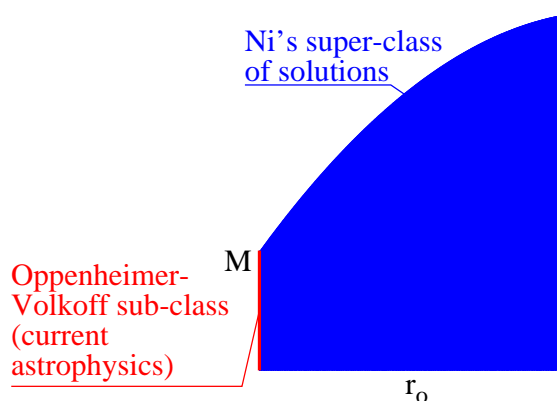


Figure 1: The scheme illustrating the extent of the super-class of the solutions of EFEs found by Ni (blue area) and its sub-class found by Oppenheimer and Volkoff (red abscissa). Parameters  $r_o$  (zero-net-gravity distance) and  $M$  (mass as the energy divided by the quadrate of light speed) continue to infinity.

### Constraining postulates; "newtonization" of general relativity

Let us imagine an astrophysicist who wants to create a model of structure of a spherically symmetric common star with the mass of  $1 M_{sun}$  and he or she starts the numerical integration of the well-known equations of stellar structure at the distance, say, of the Venus' orbit from the stellar center. He or she will obtain a model of red-giant star, but the choice of starting distance at the Venus' orbit will rule out the model of main-sequence star.

A similar problem with the starting distance also occurs in modeling of a RCO. Having the Ni's solution, one can clearly demonstrate that everybody, who starts the numerical integration of the relevant EFEs at the center of RCO, implicitly postulates that

- (i) the values of the size of  $g_{rr}$ -component of metric tensor in the interval from 0 to 1 are forbidden,
- (ii) the metrics inside the spherically symmetric material shell becomes (or has to be postulated to be) the Minkowski metrics, and
- (iii) the gravitational acceleration has to exclusively be linearly proportional to the mass of object (mass calculated as the ratio of energy,  $W$ , and quadrate of light speed,  $c^2$ , according to the well-known Einstein's formula).

Due to the above mentioned postulates, the GR in its application to RCOs was constrained - "newtonized". With the help of the Ni's solution, we can show that the upper mass limit of neutron stars is not any consequence of the GR, as generally believed, but this limit and, consequently, the black holes are the consequence of the constraint of GR.

Ni started the integration in a finite RCO-centric distance and, thus, ignored the GR-newtonizing postulates. Consequently, he obtained the solution enabling to construct a model of RCO of whatever a large mass and with the outer surface always situated above the event horizon.

## Note on spherical symmetry

We have to realize that the concept of the spherical symmetry is trivial in the Euclidean space of Newtonian physics: a spherically symmetric distribution of matter is observed as being spherically symmetric by every observer, in whatever position. However, in the curved spacetime (or curved space in a static case) in the GR, the distribution of matter which is seen and measured as spherically symmetric by the observer in its center is no longer spherically symmetric, in general, for an observer aside the center.

According to the Ni's solution, the concerning asymmetry causes that a particle inside the RCO is attracted by the RCO's outer spherical layers (outer in respect to the particle's position) away from the RCO's center. An analogy of this "outward oriented gravitational attraction" is the example of a body between the Earth and Moon, but in the gravitational domination of the latter. The body is accelerated toward the Moon, i.e. outward in respect to an observer on the Earth.

The claim about the outward accelerated particle remains true unless there is established a postulate ruling all RCOs with the outward oriented gravitational attraction out, i.e. the postulate cancelling the validity of the GR in such a case, and replacing, in fact, the EFEs.

## Question on an applicability of the Ni solution to the real objects

Since the difference between the Ni solution and O&V sub-class is caused by the postulates, no theoretical reasoning in favour or disfavour of the applicability of Ni solution to describe the real objects can decide the problem.

The decision can be done on the basis of future or already done observations of objects and phenomena predicted by using the Ni solution. Some of the objects and phenomena are outlined in the following.

## Pulsars with an extraordinary large mass

Since the mass of the Ni's objects is not limited, the pulsars with a very large mass (the mass of neutron-star-like objects is limited by the demand of density to be above the neutron drip) could be discovered if the Ni solution is applicable to the reality. Some pulsars with a relatively large mass were already discovered (e.g. J1748-2021B, J1311-3430, or B1957+20 with masses 2.5–2.9, 2.1–2.8, and 2.1–2.7  $M_{sun}$ , respectively; Watts et al., 2015). Or, Schroeder & Halpern (2014) obtained the "best fit" for the mass of pulsar J1810+1744 larger than 10  $M_{sun}$ , but they were forced to accept an unplausible irradiation efficiency of  $\sim 100$ .

Unfortunately, a discovery and publishing of a very massive pulsar is complicated with the opinion of authors and reviewers in the peer-reviewed journals that a very large mass of pulsars "must be a systematic error" because the (old) theory does not permit such a mass.

## Relativistic radiation spheres: ultra-luminous objects (quasars?)

The Ni's strategy enables to construct a super-massive object consisting of radiation - relativistic radiation sphere (RRS). Using the EFEs for the spherical symmetry (the same as O&V used) with the equation of state for the radiation,  $E = 3P$  ( $E$  is the

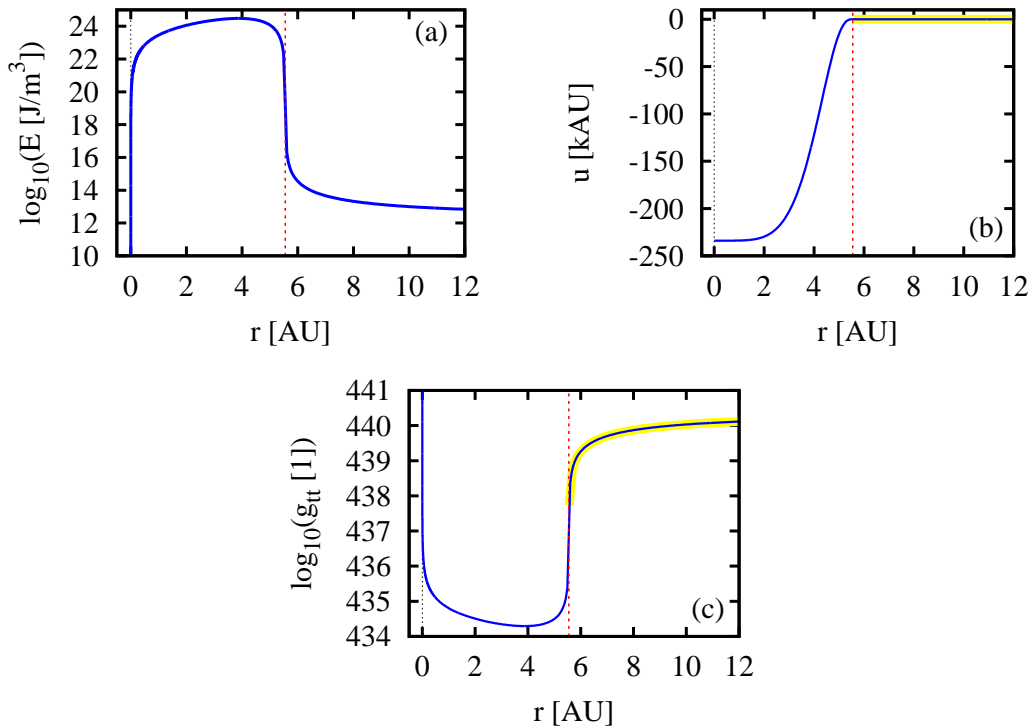


Figure 2: Behaviors of energy density,  $E$  (plot a), parameter  $u$  related to the  $g_{rr}$ -component of metric tensor (b), and  $g_{tt}$ -component of metric tensor (c) in an example of the relativistic radiation sphere. The behaviors are plotted by the solid blue curves. The fit of the behavior by the corresponding outer Schwarzschild metrics in plots b and c is shown with the thick yellow curve. The dashed red horizontal line indicates the border of the central condensation of object.

energy density and  $P$  is the pressure), we constructed the object with the behaviors of  $E$ ,  $u$ , and  $g_{tt}$  shown in Fig. 2. Quantity  $u$  is other form of  $g_{rr}$  component of metric tensor defined by  $u = r(1 + 1/g_{rr})/2$  and  $g_{tt}$  is another component of metric tensor.

In the  $E$ -behavior (Fig. 2a), we see a central condensation (CC) having the radius (5.55 AU) slightly larger than the radius of the Jupiter’s orbit in the Solar System. The total energy inside the CC equals  $W = 5.55 \times 10^{59}$  J ( $W/c^2 = 3.10 \times 10^{12} M_{sun}$ ). It is such a large energy that the object emitting the radiation, from its photosphere, with the luminosity of bright quasar (e.g. 3C 273 having luminosity about  $3.0 \times 10^{40}$  J s $^{-1}$ ; Courvoisier & Camenzind 1989; Bednarek & Calvani 1991) during the age of the universe (13.799 Gyr; Bennett et al. 2012) would spend only 2.3% of the total energy being in its CC.

At the border of the CC,  $E$  suddenly decreases about 6 orders of magnitude. Beyond the border, it is non-zero, but negligible, in a certain interval of distance, in comparison to that inside the CC, therefore the metrics outside can be well approximated with the outer Schwarzschild metrics (yellow curves in Fig. 2b and 2c). The trajectory of an object in a vicinity of the CC would practically be a Keplerian cone section, like that in the Newtonian potential generated by the central body of mass  $2.8 \times 10^8 M_{sun}$  (= mass of 3C 273; Pian et al. 2005).

The energy of the RRS is spread, in the form of radiation, to a large distance. In the distance scale of several kiloparsecs, the net energy of the RRS in form of ”corona” exceeds the energy in the CC and increases linearly with the radial distance (the distribution of energy density is shown in Fig. 3a). The quantities  $uc^2/G$  ( $G$  is the

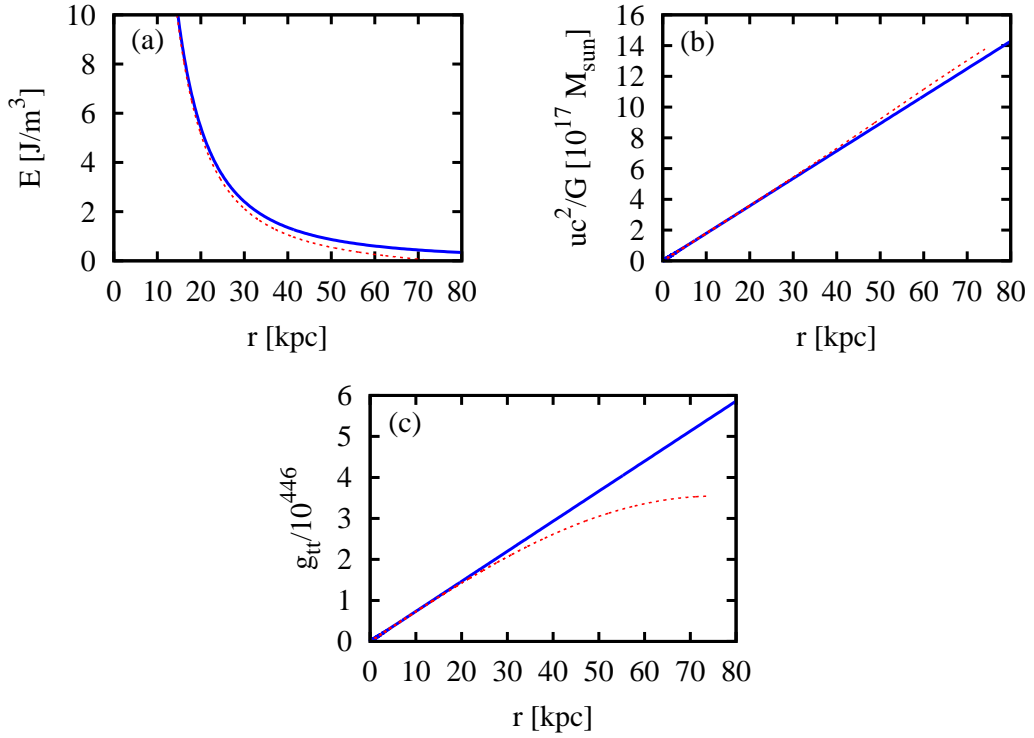


Figure 3: The same behaviors as in Fig. 2 in the scale of tens of kiloparsecs. The behaviors for the zero value of cosmological constant  $\Lambda$  are plotted with the solid blue curves and those for this constant equal to  $8 \times 10^{-44} \text{ m}^{-2}$  with the dashed red curves.

gravitational constant) and  $g_{tt}$  also linearly increase with this distance (Fig. 3b,c) in the RRS corona when the cosmological constant  $\Lambda$  is negligible. The dynamics of some objects, e.g. stars, moving around the RRS's center in the kiloparsec-scale distance is considerably different from the Keplerian motion. The gravitational effects by the corona resemble those we assign to the dark matter in a galaxy.

### Implications from the models of RRS

The above outlined model of the RRS implies the existence of some objects and phenomena:

\* an extremely high luminosity of the CC; it is possible that the CCs of RRSs are observed as quasars

\* an assumption of the super-Eddington accretion and, hence, the violation of old law of astrophysics is not necessary; the mechanism of the radiation due to an accretion disk can likely be replaced by the concept of the extreme radiation from the photosphere of the CC of RRS

\* in the universe, the RRSs could exist from the cosmological era of radiation; at the end of this era, the radiation fluid was torn into chunks, which acquired a (quasi) spherical shape due to the self-gravity and the most massive of them have survived until the present; there could be no dark age

\* the CCs of less massive RRSs can be expected to be cooled, the high-energetic photons re-combine to pairs of particle and anti-particle; if we assume an asymmetry of matter and anti-matter, a nuclear interaction of fermionic particles later occurred (are active galactic nuclei the phase of thermonuclear burning of former CCs of RRSs?)

\* using, e.g., the polytrope as the equation of state, we can construct a model of stable, quiet, RCO after the nuclear burning is gone; the large-scale, massive corona of such an object should persist; the compact object Sgr A\* in the center of Milky Way can probably be explained with the model of cold RCO

\* the predicted Keplerian orbits around the CC and analogous orbits expected around the cold RCO are observed in the case of stars orbiting the Sgr A\*

\* if there is observed the super-massive compact object in the centers of quasars and galaxies there must also be the extensive corona, which is actually observed, mainly as a galactic dark matter halo; the rotation curves of stars in the spiral galaxies, which are observed to be considerably different from the Keplerian behavior, are obviously shaped by the gravity of the corona of central compact object

### Future perspective

The work to construct the models of RRSs and RCOs using the Ni's strategy to solve the EFEs is still in its beginning. One can hope that not only qualitative, but also quantitative evaluation of the models will be done and the question on the applicability of the Ni's solution to construct the real objects will be answered. Our first outline of possible models anyway indicates that the original, geometrical GR by Albert Einstein is miraculous theory for the relativistic astrophysics.

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