Astrophysics I
Problem sheet 2

HS 2014
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Due: 30. September 2014

Excercise 2.1 Measuring the Astronomical Unit with the Venus transit

After Kepler published his third law in 1619, the distances between the planets in the Solar System were all known relative to the distance between the Earth and the Sun (1 Astronomical Unit). A measurement of one of these distances was therefore enough to determine the physical length of 1 Astronomical Unit (AU).
One of the first measurements was conducted making use of the Venus transit in 1761, as proposed by British astronomer Edmund Halley. The aim of this exercise is to follow the main steps in his derivation and to obtain our own estimate of the AU.

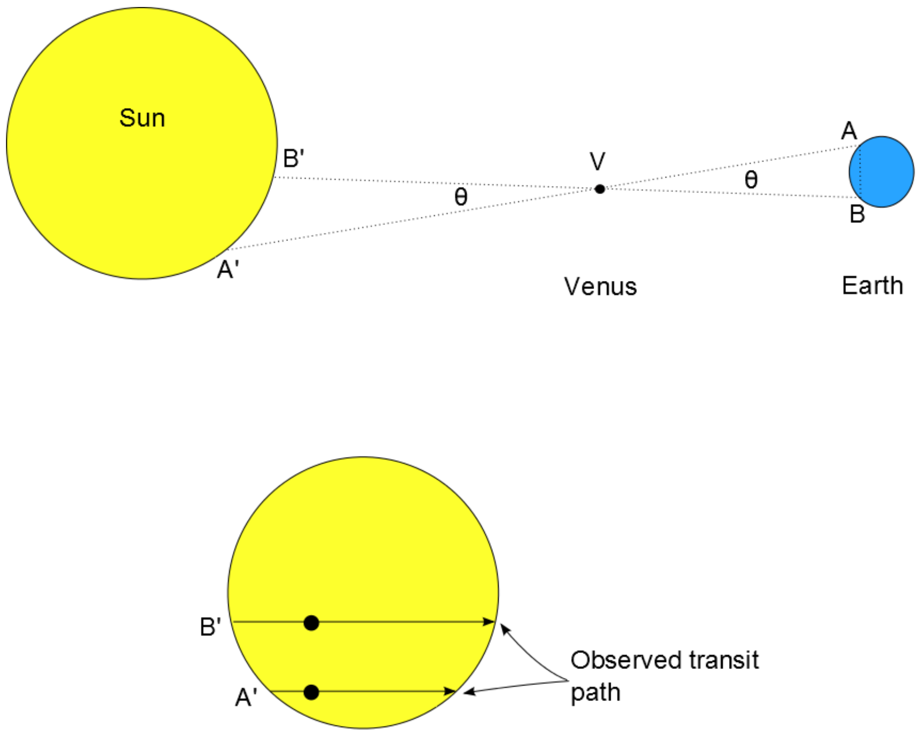

Figure 1: Geometry of Sun, Earth and Venus system. [Image credit: `http://brightstartutors.com/blog/2012/the-transit-of-venus`]

As shown in Fig. 1 the transit of Venus will be observed at slightly different positions when viewed from different points on Earth. Knowing the parallax of Venus θ_V and the distance between observers A and B the Earth-Venus distance can be calculated and therefore all distances between the planets in the Solar System. Halley proposed to infer θ_V by timing the duration of the Venus transit at two different locations on Earth A and B.
In order to estimate the AU we will use data from 2004's Venus transit observed in Cairo (A) and Durban (B).
To solve this exercise we will make the following assumptions:

1. The Earth does not rotate.

Table 1: Measurement data [credit: `http://www.phy6.org/stargaze/Svenus3.htm`]

Location	Start of transit	End of transit
Cairo (A)	5:39:09	11:04:35
Durban (B)	5:35:52	11:10:07

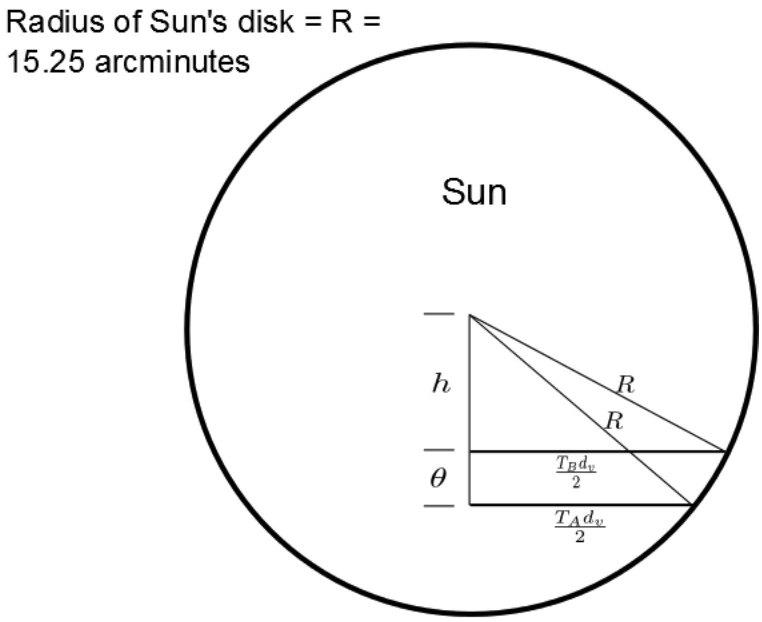

Figure 2: Geometry of Venus transit. [Image credit: `http://brightstartutors.com/blog/2012/the-transit-of-venus`]

2. The Venus transit happens when Venus is below the Earth-Sun plane.

Questions:

1. Assuming small angles, determine the angular separation between the two Venus transits θ using Fig. 2 and Tab. 1. The angular size of Sun's disk is given by $R = 15.25$ arcmin and the angular velocity of Venus is $d_{\mathrm{V}} = 0.0669$ arcsec/sec.

2. Using the angle θ calculated in the previous exercise, determine the parallax of Venus θ_{V} as a function of the parallax of the Sun θ_{S} using Fig 3.

   Hint: Relate the angle θ to the two angles β_{A} and β_{B}.

3. Assuming that the distance between the two positions A and B is $d_{\mathrm{AB}} = 5840$ km and using Kepler's third law

$$\frac{T^2}{a^3} = \text{const.} \tag{1}$$

   where a is the semi-major axis of the planetary orbit and T is its orbital period, determine the Astronomical Unit using $T_{\mathrm{Venus}} = 224$ days and $T_{\mathrm{Earth}} = 365$ days. You can work with the assumption that planets move on circular orbits around the Sun.

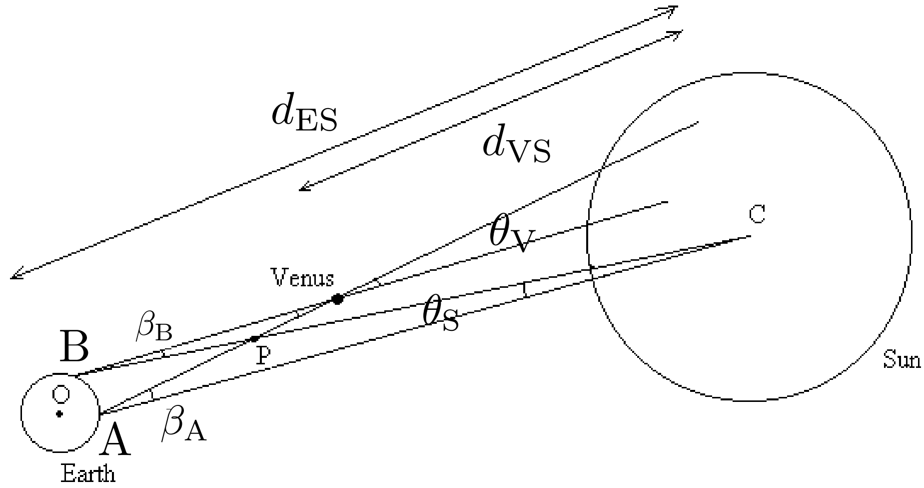

Figure 3: Effect of Solar parallax on measurement of Venus parallax. [Image credit: `http://www.vigyanprasar.gov.in/REGIONAL_MEET/north_eastern_zone/Calculating%20Astronomical%20Unit%20from%20Venus%20Transit.pdf`]

Note: The value of the AU derived in this exercise will deviate from the value discussed in class. This is due to our simplified assumptions.